

Consequences of Consumer Sales Taxes in Light of Strategic Suppliers

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Abstract

Taxes levied on retail sales are a ubiquitous form of taxation, both in the US and abroad. While considerable study has examined the economic effects of such sales taxes vis-a-vis consumer demand, surprisingly little attention has been focused on the effects up the supply chain. In this paper, we consider a parsimonious model of retail products sold in a variety of consumer markets (each of which may face different tax rates) when retailers rely on strategic suppliers for inputs in the products they sell. We find that when suppliers have and use pricing power, the imposition of sales taxes at the retail level has reverberations on supply markets – sales taxes undercut consumer demand which also makes retailers more price-sensitive, and suppliers respond to this by cutting prevailing input prices. Not only does this "soften the blow" of sales taxes on retail profit in the market (tax jurisdiction) in question, it also boosts retail profit in other markets since the retailer is able to parlay the lower input prices into greater margins therein. Besides reversing several conventional views of economic consequences of sales taxes, the results may also provide key implications for tax policy when firms operate in and care about multiple tax jurisdictions.

1. Introduction

The economic consequence of imposing sales taxes on consumer products, a long-studied and oft-debated question, has taken on greater importance in recent years with the proliferation of online sales and the resulting taxation implications. Beyond the issue of effective implementation, questions of the efficacy of introducing widespread taxation on internet purchases (the latest legislative incarnation of which is the "Marketplace Fairness Act") have brought the broader issue of sales taxes to the forefront of many public policy discussions. While most are focused on how sales taxes affect consumer behavior and how that, in turn, affects retail sellers' decisions, little (if any) attention has been paid to the consequences of such end-user taxes on input providers at the wholesale level. Starting with the notion that end-user sales subject to taxation typically run through a nontrivial supply chain before reaching consumers, this paper seeks to examine if and how such supply chain relationships are altered by taxes imposed on end users.

To elaborate, we consider a parsimonious model of a retailer selling to consumers, where consumer purchases are subject to taxation. In this model, we incorporate two distinct practical features: (i) the retailer relies on a wholesale supplier in providing goods to consumers; and (ii) the retailer sells to consumers in various markets, each of which may be subject to different tax rates. The former feature captures the notion that retail providers are rarely vertically integrated but instead rely on suppliers for their various products. The latter feature captures the idea that different states or distribution methods (e.g., online vs. in-store) are subject to different taxation despite the fact that the goods themselves are equivalent.

Our model demonstrates that these two key features work in concert to alter traditional thinking about sales taxes and their consequences. First, we show that increases in sales taxes in one jurisdiction, while stunting consumer demand and thus restricting incentives for retail supply therein, have notable reverberations in input markets. By

restricting retail margins, high sales taxes limit retailers' willingness to pay for inputs which, in turn, incentivizes price cuts at the wholesale level. Second, we show that due to this input market effect, higher sales taxes in one jurisdiction boost demand in other jurisdictions even if consumers in the jurisdictions do not overlap. The reason for this is that high taxes in one market compel lower input prices and these lower input prices incentivize greater retail supply in other, low-tax markets.

The cross-market interlinkage introduced by the nexus of differential end-user taxation yet reliance on a common input across markets has some notable implications for tax policies and firms' lobbying efforts (or lack thereof). First, a government seeking a tax increase may meet little resistance (or even tacit support) from a retailer if that retailer stands to benefit from input market price-cuts the proposed increase may engender. The paper provides precise conditions under which the multi-market retailer profits despite an increase in a market's sales tax rate. Second, one government may benefit from higher tax collections when another government imposes higher taxes even though the consumers in these markets are independent. Third, tax hikes in one jurisdiction can even boost welfare if their effect is to boost supply chain efficiency and better level retail supply across markets. These results may explain, for example, tacit support by retailers of sales taxes at bricks-and-mortar locations yet adamant and organized opposition to sales taxes for online purchases. They may also explain the tendency for governments to boost sales taxes when their tax rates are lower than others, even when income and property taxes differ vastly (i.e., it is a differential in sales tax rate, not the overall tax burden, that introduces a benefit to altering sales tax rates).

To extend the analysis and test its robustness, we examine two modeling variants. First, we consider consequences of unit (excise) taxes on the analysis, showing that it is taxation at the consumer level (not a percentage tax rate) that is the key feature, but also showing that input markets notably alter the traditional comparison of sales and unit tax methods. Second, we examine the consequences of competition, both at the retail and

wholesale level. This extension demonstrates that the key considerations identified herein persist under competition, but that increased competition in either the wholesale or retail arena mitigates the supply market effects we identify. This suggests that the input market effects should be most pronounced in practice for markets characterized by substantial supplier power and/or retailer market concentration.

This paper's findings fit into two broad and heretofore distinct streams of literature: (a) economic consequences of tax policy and (b) supply chain pricing. In terms of extant research on taxes and their economic consequences, there is of course a voluminous collection of research examining how corporate taxes affect the economic growth, income taxes affect labor incentives, and sales taxes affect retail firm behavior. Most closely related to the present study is the latter stream that hones in on consumer sales taxes. Much of the focus there is on if and how such taxes affect retailer behavior and whether imposing \$1 of retail taxes actually leads to an increase of out-of-pocket consumer cost of \$1. This tax incidence literature notes that the chilling effect of taxes on consumer demand may restrict retail quantities and boost consumer out-of-pocket price beyond just the tax imposed. Such "overshifting" can arise both when taxes are imposed on firms and when they are imposed on consumers. Studies have shown that such overshifting can be mitigated by, among other things, retail competition and excess capacity (Anderson et al. 2001a; Anderson et al. 2001b; Marion and Muehlegger 2011).

What is notably absent from this extensive research on taxation policy, however, is an examination of the upstream consequences of sales taxes when the supply chain for retail goods is imperfectly coordinated, the focus herein. A consequence is that our paper demonstrates that conventional results that firm profits are always dampened and that welfare is necessarily reduced by higher taxes may well be reversed when supply chain efficiency gains are taken into account. Additionally, by considering an uncoordinated supply chain subject to a dominant supplier, we show that overshifting may be minimized due to the manner in which retail sales taxes convince suppliers to cut their prices. In fact,

when firms operate in multiple consumer markets, the supply market consequence can actually lead to lower retail prices in more than one market. These results may provide some support for the empirical evidence suggesting undershifting – tax increases of \$1 leading to out-of-pocket consumer cost increases less than \$1 – is prevalent in many industries (e.g., Besley and Rosen 1999; Poterba 1996).

Given the critical role of multiple retail jurisdictions served by a retailer, our study is naturally tied to, and offers implications for, the nascent literature examining how differential tax rates between in-store and online purchases affect consumers and firms who each operate in both markets (e.g., Baugh et al. 2014; Goolsbee and Zittrain 1999; Hoopes et al. 2014). More broadly, the multi-market emphasis is in line the recent work of Hamilton (2008) who examines how taxes imposed in some retail markets affect other retail markets of multiproduct firms. In Hamilton (2008), the interlinkage comes about due to complementarities in goods and other intrinsically-tied consumer demand; here, in contrast, the interlinkage arises due to reliance on the same input for multiple retail markets with independent consumer demand.

The second key stream of literature this paper builds upon concerns upstream markets and their behavior. At the core of the bulk of these studies (and this paper too) is the inherent conflict of interest in pricing. Starting with the seminal work of Spengler (1950), many have examined distortions introduced by above-cost pricing by suppliers and retail firm efforts to alleviate such distortions (for thorough and excellent reviews of the literature on supply chain pricing, see Katz 1989 and Lariviere 1998). Complicating matters are strategic efforts by suppliers to solidify excessive input prices, including sabotaging downstream investments (Pal et al. 2012) and even self-sabotage (Sappington and Weisman 2005).

Existing literature has shown that concerns over supplier pricing can explain a variety of practices, including the introduction of direct sales by a supplier (Tsay and Agrawal 2004), cost-plus transfer pricing by a firm (Arya and Mittendorf 2007), product

returns policies (Pasternack 1985), quantity flexibility or revenue-sharing arrangements (Tsay 1999; Cachon and Lariviere 2005), and propping up loss-leader products (Arya and Mittendorf 2011). This paper adds taxation at the retail level as an additional consideration for supply chains, demonstrating that the imposition of sales taxes in one tax jurisdiction can have important ramifications for supplier pricing which, in turn, has ramifications for retail pricing and supply in other jurisdictions.

The paper proceeds as follows. Section 2 presents the basic model. The results are presented in Section 3: the retail market equilibrium under sales taxes is identified in 3.1; the benchmark case of a vertically-integrated supply chain is presented in 3.2; the consequences of strategic supplier pricing are identified in 3.3; an extension to unit rather than *ad valorem* sales taxes is studied in 3.4; and the effects of competition, both at the retail and wholesale level, are examined in 3.5. Section 4 discusses implications and concludes the paper.

2. Model

A firm relies on a supplier for a key input of products it provides in various consumer markets. In market i , consumer purchases are subject to an *ad valorem* sales tax, with the tax rate denoted $t_i \geq 0$. The distinct markets can reflect different geographic areas with varying tax rates (cities, counties, or states); different distribution methods that face different tax regimes (online vs. in-store); and/or different consumer uses taxed differently (food purchased for dine-in vs. to-go). To highlight the role of sales taxes, we presume that beyond any differential tax rates, the markets are identical and independent. In particular, (inverse) consumer demand in market i is $\hat{p}_i = a - q_i$, where q_i reflects the quantity of units sold in market i and \hat{p}_i reflects the consumer's out-of-pocket price paid for each unit. The consumer's out-of-pocket cost consists of stated retail price, p_i , plus taxes, $t_i p_i$, i.e., $\hat{p}_i = p_i[1 + t_i]$.

The monopolist supplier produces the inputs at unit cost v , $v \geq 0$, and sets a unit input (wholesale) price of w , $w \geq 0$. As is typically the case, consumer taxes are only levied at the retail level and not wholesale level. Thus, the retail firm's out-of-pocket cost for each unit of input is w ; denote any subsequent costs to convert and sell each input by c , $c \geq 0$. Given this formulation, the supplier's and firm's profits in market i are $\Pi_i^s = [w - v]q_i$ and $\Pi_i = \left[\frac{a - q_i}{1 + t_i} \right]q_i - wq_i - cq_i$, respectively. Similarly, the taxes collected in market i are $T_i = t_i \left[\frac{a - q_i}{1 + t_i} \right]q_i$, and total welfare (surplus) in market i is $\Psi_i = [a - q_i]q_i - vq_i - cq_i + \frac{q_i^2}{2}$.

The sequence of events is as follows:

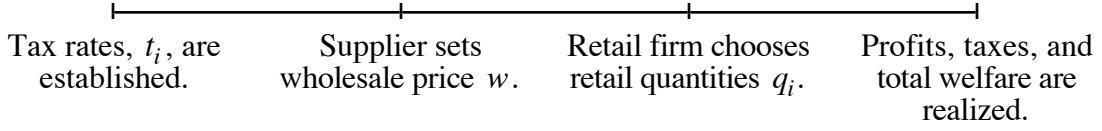


Figure 1: Timeline

Denote the number of consumer markets the retailer serves in equilibrium by n , $n \geq 2$. Given the prevailing market tax rates, $\{t_1, \dots, t_n\}$, let \bar{t} denote the mean tax rate and t^{\max} denote the maximum tax rate. Given this, we presume consumer demand in the markets is sufficiently large to ensure interior solutions; in particular, $a > \left[\frac{(1 + t^{\max})(1 + \bar{t})}{1 - t^{\max} + 2\bar{t}} \right] [c + v]$.

Using this basic setup as a backdrop, we examine how (changes in) sales taxes affect profits, retail sales, tax collections, and welfare. The analysis is conducted with a focus on two key features novel to the setting: (i) retail sales occur in different consumer markets facing possibly different tax rates; and (ii) retail sales require wholesale purchases.

3. Results

In deriving the equilibrium outcome, we work backwards in the game beginning with the retail market equilibrium.

3.1. Retail Market Outcome

For a given prevailing wholesale price, w , the firm chooses retail quantities, q_i , $i = 1, \dots, n$, to maximize its total profit, $\Pi = \sum_{i=1}^n \Pi_i$. The first-order condition of the firm's problem yields the retail market quantities, consumer (out-of-pocket) prices, and retail prices:

$$q_i(w) = [1/2] [a - (1 + t_i)(c + w)], \hat{p}_i(w) = a - q_i(w) = [1/2] [a + (1 + t_i)(c + w)], \text{ and} \\ p_i(w) = \hat{p}_i(w) / [1 + t_i] = [1/2] [a / (1 + t_i) + c + w], \quad i = 1, \dots, n. \quad (1)$$

The outcome in (1) reflects the usual properties of retail output – it is increasing in consumer demand (a) and decreasing in cost ($c + w$). The new feature here is that retail quantities are also reduced by higher sales tax rates. Roughly stated, the tax-imposed cost on the consumer is borne by the retail seller in the form of an increase in the "effective" marginal cost (see, e.g., Anderson et al. 2001). Viewed in terms of prices, consumers' out-of-pocket payments are of course boosted by an increase in the tax rate. This reduces their willingness to pay and, as a consequence, leads to a cut in the retail price – this is captured by the term $a / [1 + t_i]$ in $p_i(w)$.

The result of this retail outcome is tax collections in market i of:

$$T_i(w) = t_i \left[\frac{a - q_i(w)}{1 + t_i} \right] q_i(w) = \frac{[a^2 - (1 + t_i)^2(c + w)^2] t_i}{4[1 + t_i]}. \quad (2)$$

The retail firm's profit across all markets, denoted $\Pi(w)$ is:

$$\Pi(w) \equiv \Pi|_{q_i = q_i(w)} = \sum_{i=1}^n \frac{[a - (1 + t_i)(c + w)]^2}{4[1 + t_i]}. \quad (3)$$

Finally, total welfare given the retail equilibrium, $\Psi(w)$, is as follows with σ_t^2 , $\sigma_t^2 = \sum_{i=1}^n [t_i - \bar{t}]^2 / n$, denoting the tax-rate variance:

$$\Psi(w) \equiv \sum_{i=1}^n \Psi_i|_{q_i = q_i(w)} = \frac{3na^2}{8} - \frac{n[c + v] \left[2a\{3 + \bar{t}\} - (c + w)\{3 + 2\bar{t} - \bar{t}^2 - \sigma_t^2\} \right]}{8}. \quad (4)$$

Given these outcomes, the paper's focus is on how supply market effects alter the traditional views of sales taxes. To do so most clearly, we first present a benchmark case where supply market effects are absent.

3.2 Insourcing Benchmark

Say the retail firm makes its inputs in-house (at cost v), and does not need to rely on a strategic supplier for wholesale goods. Alternatively, one can view this as the outcome when there is an integrated supply chain or perfectly competitive input market. In each interpretation, the outcome corresponds to the equilibrium in Section 3.1 with $w = v$. Thus, using (1) through (4), with $w = v$, provides the solution in the insourcing case. This benchmark equilibrium yields the prevailing views of the consequence of higher sales taxes as summarized in the first proposition. (All proofs are provided in the appendix.)

PROPOSITION 1. Under insourcing, an increase in sales tax in market i

- (i) reduces the firm's profit, i.e., $\frac{d\Pi(v)}{dt_i} < 0$;
- (ii) can increase or decrease tax revenues in market i while leaving tax revenues in other markets unchanged; and
- (iii) increases consumer "out-of-pocket" price in market i while leaving consumer prices in other markets unchanged, i.e., $\frac{d\hat{p}_i(v)}{dt_i} > 0$ and $\frac{d\hat{p}_j(v)}{dt_i} = 0$ for $j \neq i$; and
- (iv) decreases welfare, i.e., $\frac{d\Psi(v)}{dt_i} < 0$.

The benchmark proposition confirms the traditional thinking about sales taxes. First, due to the chilling effect on consumer demand, higher sales taxes undercut firm profits (Proposition 1(i)). Second, in terms of tax collections, higher taxes entail a tradeoff of increasing the per-unit haul by the government and reducing the retail quantities subject to taxation, and can thus increase or decrease tax revenue depending on the circumstances (Proposition 1(ii)). Third, by cutting demand, tax increases ultimately increase consumers'

out-of-pocket costs in the market they are imposed but have no effects on consumer prices in other markets (Proposition 1(iii)). And finally, the net effects of sales tax increases is to reduce overall welfare, even if they boost tax collections. This speaks to the general feeling that the net economic repercussions of tying tax collections to underlying economic activities renders them counterproductive.

With this basic benchmark as a backdrop, we now consider how the consideration of wholesale supply alters traditional views.

3.3 Outsourcing to a Strategic Supplier

To determine the outcome with a strategic supplier, we return to the retail market equilibrium in section 3.1, and step back to consider the supplier's choice. The supplier's total profit for a given wholesale price, denoted $\Pi^s(w)$, is:

$$\Pi^s(w) \equiv \sum_{i=1}^n \Pi_i^s \Big|_{q_i = q_i(w)} = \frac{n}{2} [w - v] [a - (1 + \bar{t})(c + w)]. \quad (5)$$

Maximizing (5) with respect to w reveals the supplier's equilibrium price, w^* : $w^* = \frac{1}{2} \left[\frac{a}{1 + \bar{t}} - c + v \right]$. The wholesale price has some intuitive features: it is increasing in the supplier's cost (v), increasing in consumer (and thus retailer) demand (a), and decreasing in the costs of retail delivery (c). And, just as tax rates suppressed consumer willingness to pay and compelled lower retail prices, the same effect transfers up the vertical supply chain compelling lower input prices too (i.e., w^* is decreasing in \bar{t}). Notice it is the average tax rate across markets that proves crucial to the supplier – after all it is concerned equally with input procurement in the aggregate, not just in a single market. The tax effect, that w^* is decreasing in \bar{t} , will prove critical and reflects the fact that the supplier must be particularly careful in squeezing retail margins if such margins are already razor thin. Using w^* in (1) and (3) yields the equilibrium outcome and firm profits, respectively, with a strategic supplier, as summarized in the following lemma.

LEMMA 1. With a strategic supplier, the input price, the firm's production decision, and its profits are as follows:

$$w^* = \frac{1}{2} \left[\frac{a}{1+\bar{t}} - c + v \right]; \quad q_i^* = \frac{a}{2} - \frac{[1+t_i][a + (1+\bar{t})(c+v)]}{4[1+\bar{t}]}; \text{ and}$$

$$\Pi^* = \frac{1}{4} \left[a^2 \sum_{i=1}^n \left(\frac{1}{1+t_i} \right) - \frac{n}{4} \left(\frac{3a^2}{1+\bar{t}} + [c+v][2a - (1+\bar{t})(c+v)] \right) \right].$$

With this equilibrium outcome in tow, we now revisit the benchmark results in Proposition 1 in the presence of a supplier, starting with an examination of firm profits.

3.3.1 FIRM PROFITS

The first and most fundamental result about the effect of sales taxes on economic activity is that they undermine retail firm profitability. That is, sales taxes stifle consumer demand which, in turn, shrinks retail margins and profitability. As may be inferred from Lemma 1, there is a mitigating feature when retail firms rely on suppliers. Though sales taxes do have a demand-side harm to the retailer, they also offer a supply-side benefit. By heightening the sensitivity of the retailer's own demand for inputs, sales taxes force supplier concessions. This is confirmed by noting that $\frac{dw^*}{dt_i} = -\frac{a}{2n[1+\bar{t}]^2} < 0$ – an increase in t_i boosts the average tax value and, to that extent, disciplines wholesale price. The result is summarized in Proposition 2.

PROPOSITION 2. An increase in market i 's tax rate decreases the supplier's wholesale price, i.e., $\frac{dw^*}{dt_i} < 0$.

In other words, Proposition 2 demonstrates a silver lining of higher tax rates – though they reduce retail demand, they also reduce supplier markups. As may be expected, the former effect outweighs the latter in market i . Lest one think the supplier pricing effect is merely a second-order one, however, it is worth noting that the cut in supplier prices is put in effect for all inputs and, thus, has spillover to other output markets. As a result,

while higher taxes in market i will indeed reduce the firm's profit in that market, they will also boost profits in other markets by reducing the prevailing input prices for the goods sold in them. This effect, in fact, can make it such that higher taxes in one market can actually increase the retail firm's profit.

PROPOSITION 3. An increase in sales tax in market i increases the firm's profit for $t_i > t_i^*$, where t_i^* is the unique t_i -value that solves $t_i = f(\bar{t})$,

$$f(\bar{t}) = \left[\frac{3}{4(1+\bar{t})^2} + \left(\frac{c+v}{2a} \right)^2 \right]^{-1/2} - 1.$$

The proposition presents a counterintuitive result but one that stresses the key point that supply chain effects of sales taxes are both subtle and potentially critical. The reasoning behind the result is that when circumstances are such that market i is (or becomes) a relatively low-profit market due to taxes imposed in it, the dampening effect on demand of a tax increase in that market is outweighed by the boost in retailer margins in its other, more profitable (lower tax), markets.

In effect, the subtlety identified herein is that because markets naturally face different tax rates, a retail firm that operates in several of them is cognizant that changes in tax rates in some markets naturally have ramifications for other markets due to their effects on supplier prices. The next figure provides a graphical depiction of the proposition's result. In particular, the figure plots firm profit as a function of the tax rate in market i for a crisp case: $c = v = 0$ and $t_j = t$ for $j \neq i$. The convexity of the profit function reflects the dampened consumer demand vs. supplier concessions tradeoff, with the low point representing the precise cutoff provided in the proposition. Intuitively, at higher t_i -values, market i is already not so profitable and, thus, dampened demand there is less consequential relative to the benefit obtained from supplier concessions in the relatively more profitable (lower tax) markets.

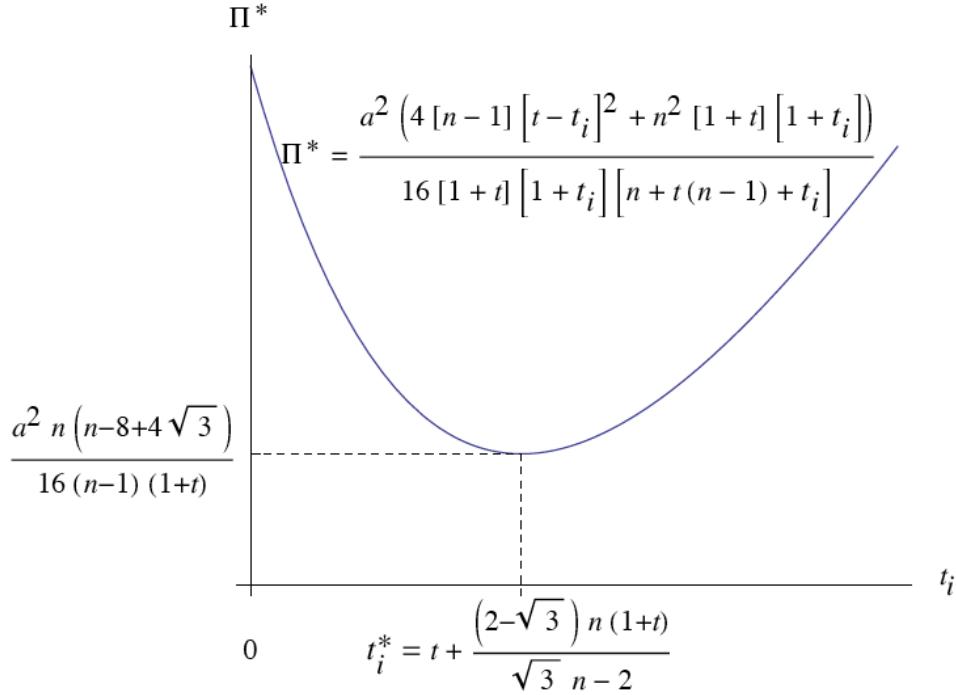


Figure 2: Firm Profit as a Function of Sales Tax Rate in Market i .

To highlight the importance of considering multiple markets and varied tax rates among them, note that the result in Proposition 3 is disabled if one considers tax changes that are applied uniformly in all markets rather than determined separately in separate markets (or, alternatively, if the firm only operates in one market).

PROPOSITION 4. An increase of Δ in the sales tax rate of each market reduces the firm's profit, i.e., $\frac{d\Pi^*}{d\Delta} < 0$.

Propositions 3 and 4 together imply that what makes the effect of local tax policy so delicate is that firms subject to it are involved in separate tax jurisdictions simultaneously, and due to supplier effects these markets and their taxes are inextricably linked even if their governance is done independently. The results also suggest that while retailers would be wise to fight sales tax increases in low-tax jurisdictions (e.g., online sales), they may not be expected to fight sales tax increases in others as vehemently. That is, it is tax rate

differentials that a firm finds helpful, so changes that create or exacerbate such differentials can be particularly helpful for a retail firm. Next, we examine the effects of changes in tax rates on tax collections.

3.3.2 TAX REVENUES

A second fundamental result about the economic consequences of sales taxes is that imposing tax increases introduces the two competing effects on tax collections: higher per-unit collections ($t_i p_i$) but lower sales quantities (q_i). As confirmed in Proposition 1(ii), in the insourcing case, this tradeoff leads to equivocal results in market i when it comes to the effect of tax rates on tax collections; all other markets are unaffected by a change in market i reflecting a lack of any intrinsic interaction.

With strategic input supply in play, two considerations are added to this tradeoff. The first is that the effect of higher tax rates on retail sales volume is itself mitigated by reductions in supplier prices. That is, though higher taxes in market i do reduce sales volume in market i , the extent of this sales volume reduction is offset by the reduction in supplier prices. This means that, all else equal, higher taxes are more likely to increase collections. The second effect of considering supplier pricing is that it introduces spillover in tax collections across markets. In particular, increasing tax rates in market i has no effect on underlying consumer demand in market j , but it does introduce changes in the prevailing prices for inputs in market j . This effect, in turn, means consumer purchases and tax collections are both boosted in market j .

More formally, tax collections in market i equal $t_i p_i^* q_i^* = \left[\frac{t_i}{1 + t_i} \right] [a - q_i^*] q_i^*$. As t_i increases, the term $t_i / [1 + t_i]$ increases. The term $[a - q_i^*] q_i^*$ is concave in q_i with a maximum at $a / 2$. From Lemma 1, $q_i^* < a / 2$ and, as expected:

$$\frac{dq_i^*}{dt_i} = -\frac{a}{4n[1 + \bar{t}]^2} \left[n - 1 + \sum_{j=1, j \neq i}^n t_j \right] - \frac{c + v}{4} < 0.$$

Given the above, $[a - q_i^*]q_i^*$ is decreasing in t_i and, hence, tax collections in market i can either increase or decrease with the tax rate in the market. The precise condition under which tax collections increase in market i are provided in the ensuing proposition. Notice that this condition also implies that tax collections increase in all markets. After all, in market j , $j \neq i$, an increase in t_i is sure to boost q_j^* (and, hence, $[a - q_j^*]q_j^*$) due to the lowering of w^* : $\frac{dq_j^*}{dt_i} = \frac{a[1 + t_j]}{4n[1 + \bar{t}]^2} > 0$. The proposition makes use of the expression for equilibrium tax collections, T_i^* , $i = 1, \dots, n$, which, given Lemma 1, can be written as:

$$T_i^* = \frac{t_i[1 + t_i]}{16[1 + \bar{t}]} \left[\frac{a^2(1 - t_i + 2\bar{t})(3 + t_i + 2\bar{t})}{(1 + \bar{t})(1 + t_i)^2} - [c + v][2a + (1 + \bar{t})(c + v)] \right]. \quad (6)$$

PROPOSITION 5. An increase in sales tax in market i increases tax revenues in every market for $t_i < t_i^T$, where t_i^T is the unique t_i -value that solves $g(t_i, \bar{t}) = 0$,

$$g(t_i, \bar{t}) = a^2 \left[2t_i(1 + t_i)^3 + n(1 + \bar{t})(3 - 4t_i - 5t_i^2 - 2t_i^3 + 8\bar{t} + 4\bar{t}^2) \right] - [1 + t_i]^2 [1 + \bar{t}] [c + v] \left[-2at_i(1 + t_i) + n(1 + 2t_i)(1 + \bar{t})(2a + (1 + \bar{t})(c + v)) \right].$$

The next figure presents a visual of the key forces in Proposition 5 for our continuing example ($c = v = 0$ and $t_j = t$ for $j \neq i$). The left-hand panel shows how market i 's tax rate affects tax collections in that market; and the right-hand panel shows how market i 's tax rate affects tax collections in other markets.

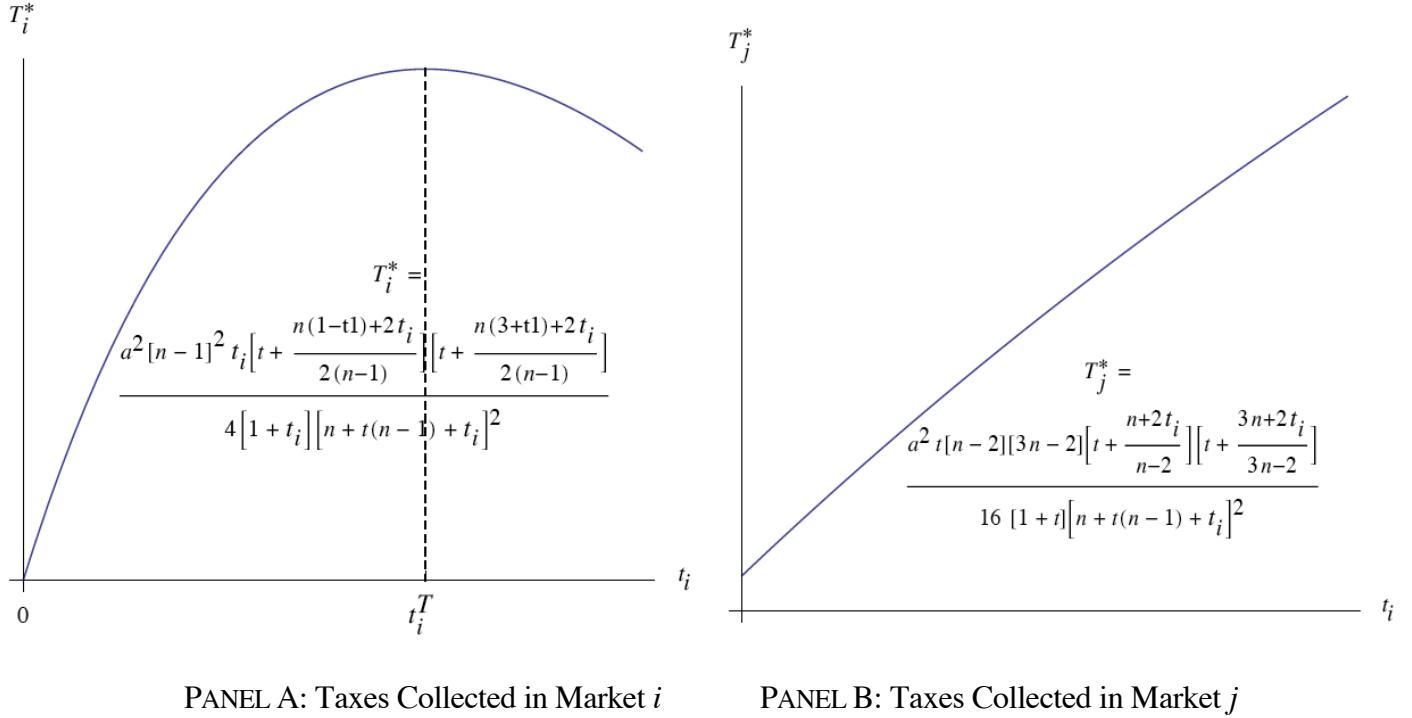


Figure 3: Tax Collections as a Function of Sales Tax Rate in Market i .

Putting the result in other words, supplier effects can not only alter the tax collection consequences of higher tax rates for the market on which they are imposed but can also positively spill over to tax collections in other markets. For this reason, one can naturally envision a circumstance where one jurisdiction encourages tax increases in another which, in turn encourages others to do the same. While the overall effects can be damaging, the "prisoners' dilemma" sort of relationship that arises amidst otherwise independent tax authorities is worth noting. With this in mind, we next consider the consequences for consumers and overall welfare.

3.3.3 CONSUMERS AND WELFARE

Turning to the final key conclusions of the benchmark case, consider how supplier-pricing effects alter the traditional view of how changes in sales tax rates affect overall welfare. Traditional views suggest that levying additional government taxes through an interlinkage with retail sales increases out-of-pocket costs for consumers (Proposition

1(iii)), and the costs to consumers reach beyond just the amount of tax collected but also brings about artificially restricted retail quantities which, in turn, harms overall welfare (Proposition 1(iv)).

The added wrinkle here is that increasing sales tax in one market reduces the prevailing input price, and this has ramifications for sales in all markets. In particular, using the prevailing retail rates, note that $\frac{d\hat{p}_i^*}{dt_i} = \frac{a[n-1+n\bar{t}-t_i]}{4n[1+\bar{t}]^2} + \frac{c+v}{4} > 0$, i.e., tax increases in market i continue to harm consumers in that market despite input price cuts. On the other hand, $\frac{d\hat{p}_j^*}{dt_i} = -\frac{a[1+t_j]}{4n[1+\bar{t}]^2} < 0$ reveals that higher tax rates in market i reduce out-of-pocket costs for consumers in other markets. This effect arises thanks to the lower input price that is shared across all markets.

PROPOSITION 6. An increase in the sales tax rate in market i , increases consumer price in market i and decreases consumer prices in all other markets, i.e., $\frac{d\hat{p}_i^*}{dt_i} > 0$ and $\frac{d\hat{p}_j^*}{dt_i} < 0$ for $j \neq i$.

The fact that tax hikes in one market hurt consumers there but prove to be a boon for other consumers means that welfare effects of tax changes may be more nuanced than previously thought. To be precise, using w^* in (4) reveals the relationship between equilibrium welfare and tax rates, as in (7).

$$\Psi(w^*) \equiv \Psi^* = \frac{na^2[7+14\bar{t}+7\bar{t}^2-\sigma_t^2]}{32[1+\bar{t}]^2} - \frac{n[c+v]\left[2a\left\{7+10\bar{t}+3\bar{t}^2+\sigma_t^2\right\} - (c+v)(1+\bar{t})\left\{7+6\bar{t}-\bar{t}^2-\sigma_t^2\right\}\right]}{32[1+\bar{t}]} \quad (7)$$

Taking the derivative of $\Psi(w^*)$ with respect to t_i reveals the subtle relationship between a jurisdiction's tax rate and overall welfare as formalized in the next proposition.

PROPOSITION 7. An increase in the sales tax rate in market i , increases welfare if $t_i < t_i^\Psi$, where t_i^Ψ is the unique t_i -value that solves $h(t_i, \bar{t}, \sigma_t^2) = 0$:

$$h(t_i, \bar{t}, \sigma_t^2) = a^2[(\bar{t} - t_i)(1 + \bar{t}) + \sigma_t^2] - [1 + \bar{t}][c + v] \times \\ \left[a\{3 + \bar{t}(4 + \bar{t}) + 2t_i(1 + \bar{t}) - \sigma_t^2\} - (3 - t_i)(1 + \bar{t})^2(c + v) \right]$$

The intuition behind the result and, in particular, the fact that tax rate hikes can actually increase welfare, stems from the positive spillover effects of taxes in one market to demand in another and the concavity of welfare in each market. The inherent concavity of welfare with respect to retail quantities simply reflects that while consumers in a market highly value a certain amount of retail supply, they do also exhibit satiation. As a result, welfare is maximized by a degree of consumption smoothing across markets: society is better off when all markets get a basic level of supply than if some markets are flush with goods and others have none. Because of this, a tax rate hike can prove helpful if it arises in a market already flush with goods available to consumers (a market with well-below average taxes). It proves helpful because the harmful effects of a cut in retail provision in that market are minor since those consumers are sufficiently satiated, but the cut in wholesale prices it engenders helps open supply to otherwise starved markets. This intuition, that welfare is boosted when tax hikes arise in very low tax areas, is supported by the cutoff representation in the proposition. The next figure reiterates the point by showing how taxes in one market affect both out-of-pocket consumer costs and overall welfare.

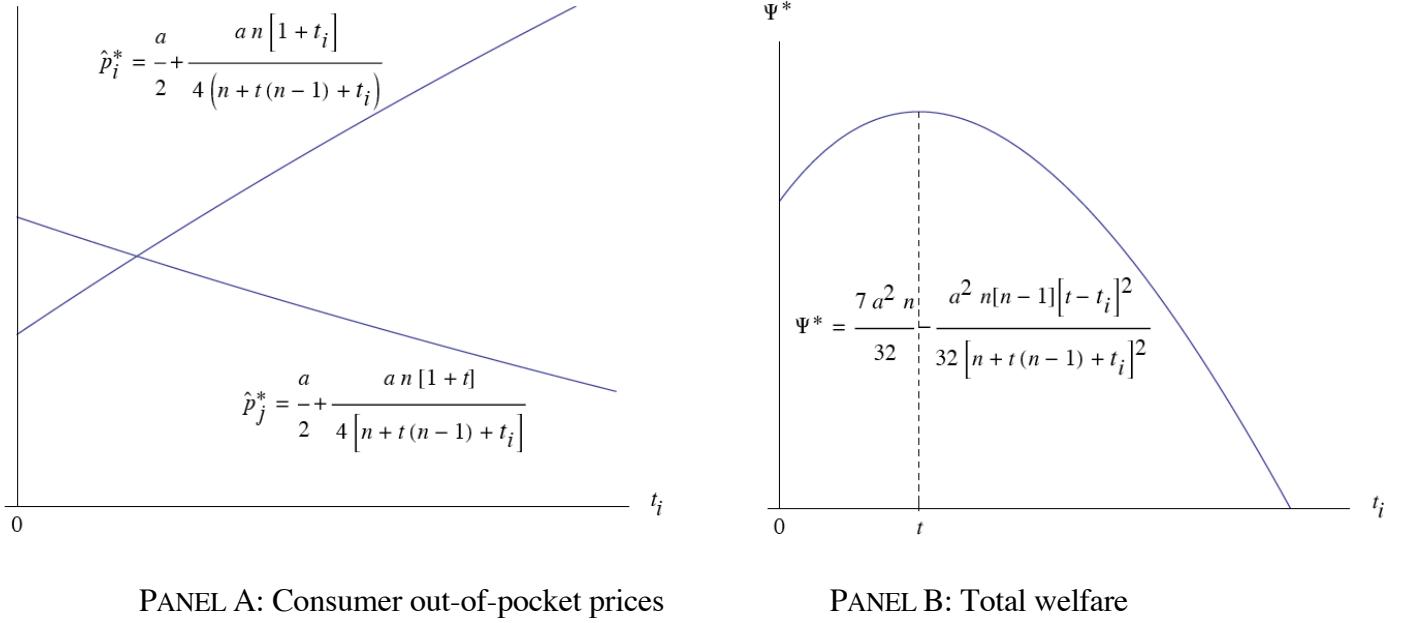


Figure 4: Welfare Consequences of Changes in Sales Tax Rate in Market i .

In the subsequent subsections, we offer extensions to the primary results in order to both test their robustness and offer additional implications. We begin with a consideration of the other primary form of retail taxation.

3.4 Ad Valorem vs. Unit Taxes

The emphasis thus far has been on the consequences of *ad valorem* sales taxes on supplier behavior and the concomitant effects for retail markets. This emphasis reflects the practical reality that most consumer sales taxes are tied to retail prices. That said, there are also circumstances where sales taxes are levied as a particular amount per unit (rather than a percentage of price), such as many gasoline, cigarette, and alcohol taxes. To examine our results under such a unit tax and revisit comparisons of economic consequences of *ad valorem* vs. unit taxes, we next repeat our analysis under a unit tax, denoting the per-unit tax in market i by t'_i and the mean unit tax by \bar{t}' . Relegating the details to the appendix, the next lemma presents the equilibrium outcome under a unit tax regime.

LEMMA 2. Under unit taxes, the supplier's input price, the firm's production decision, and its profits are as follows:

$$w' = \frac{1}{2}[a - \bar{t}' - c + v]; q_i' = \frac{1}{4}[a - 2t_i' + \bar{t}' - c - v]; \text{ and}$$

$$\Pi' = \frac{1}{4} \sum_{i=1}^n t_i'^2 + \frac{n}{16} \left(a^2 - 2a\bar{t}' - 3\bar{t}'^2 - [c+v][2(a-\bar{t}')-c-v] \right).$$

Using the equilibrium outcome in Lemma 2, the next proposition presents the unit tax analogs to Propositions 2 and 3.

PROPOSITION 8.

- (i) An increase in market i 's unit tax rate decreases the supplier's wholesale price, i.e., $\frac{dw'}{dt_i'} < 0$.
- (ii) An increase in the unit tax rate in market i increases the firm's profit for $t_i' > [a + 3\bar{t}' - c - v] / 4$.

Note from Proposition 8 that the fundamental forces at work under *ad valorem* taxes remain in play under unit taxes. That is, by lowering consumer valuation of each retail unit purchased, unit sales taxes shrink consumer demand and increase the sensitivity of the retailer's input purchase to the wholesale price. This natural compression of retail margins, in turn, compels the supplier to cut its chosen wholesale price to ensure sufficient demand (Proposition 8(i)). Despite unit sales taxes in market i undercutting retail profit in that market, the retailer gains spillover effects in other markets due to the wholesale price cut. Provided the other markets present sufficient relative profit potential, this wholesale price benefit can outweigh the loss of demand in market i , and the retailer can again benefit from unilateral hikes in tax rates (Proposition 8(ii)).

The natural follow-up question, and one routinely asked, is to consider how outcomes compare between *ad valorem* and unit taxes. In particular, suppose each market

is characterized by an *ad valorem* sales tax as in the original model formulation, and market i shifts from its *ad valorem* sales tax of t_i to unit tax of t_i'' in such a way as to keep the consumers in that market unaffected. In our setting, this corresponds to the equilibrium consumer out-of pocket price in market i remaining unchanged. With consumers in that market left indifferent by the switch, how is the retail firm affected? The next proposition demonstrates the subtlety that input market effects bring to the question.

PROPOSITION 9. Consider a shift from an *ad valorem* sales tax to a unit tax in market i that results in the same out-of-pocket cost for market i consumers. In this case, denoting the average tax rates in the other markets by \hat{t} , i.e., $\hat{t} = \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^n t_j$,

- (i) if the firm insources, it prefers the unit tax; while
- (ii) if the firm outsources, it may prefer the *ad valorem* tax. In particular, for $c = v = 0$, the *ad valorem* tax is preferred for $t_i > \tilde{t}_i = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$, where:

$$A = -4n^4[1 + \hat{t}]^2 - 4[1 + 2\hat{t}]^2 + 24n[1 + 3\hat{t} + 2\hat{t}^2] + 3n^3[9 + 17\hat{t} + 8\hat{t}^2] - n^2[44 + 99\hat{t} + 52\hat{t}^2];$$

$$B = 8\hat{t}[1 + 2\hat{t}]^2 + 4n^4[1 + \hat{t}]^2[3 + 4\hat{t}] - 56n\hat{t}[1 + 3\hat{t} + 2\hat{t}^2] - n^3[23 + 135\hat{t} + 192\hat{t}^2 + 80\hat{t}^3] + n^2[9 + 143\hat{t} + 284\hat{t}^2 + 144\hat{t}^3]; \text{ and}$$

$$C = -16n^4\hat{t}[1 + \hat{t}]^3 - 4\hat{t}^2[1 + 2\hat{t}]^2 + 32n\hat{t}^2[1 + 3\hat{t} + 2\hat{t}^2] + 2n^3[-1 + 11\hat{t} + 60\hat{t}^2 + 80\hat{t}^3 + 32\hat{t}^4] - n^2[-1 + 6\hat{t} + 100\hat{t}^2 + 192\hat{t}^3 + 96\hat{t}^4].$$

Proposition 9(i) confirms the well-known traditional comparison (e.g., Anderson et al. 2001b). Absent input market effects, *ad valorem* taxes can achieve the same retail market equilibrium as unit taxes while shifting more of the surplus away from the retail firm in favor of the government. The reason for this is that unit taxes add one dimension of consumer price-sensitivity – each unit costs a fixed amount more for consumers which, in turn, restricts retail supply. *Ad valorem* taxes, on the other hand, introduce two dimensions of consumer price-sensitivity – each unit costs more for consumers which, in

turn, restricts retail supply; the restricted retail supply ups retail price which then further adds to the tax-imposed additional consumer cost. By adding this second dimension of consumer price-sensitivity, *ad valorem* taxes add more "punch" to the destruction of retailer margins.

Proposition 9(ii) confirms that even this standard result can reverse in the presence of input market considerations. The reasoning for the flip is precisely because of the added punch *ad valorem* taxes add to shrinking retailer margins: this rapid shrink in a retailer's margin leads to more rapid cuts in the wholesale price set by the supplier. That is, denoting the wholesale price with the unit tax by w^u , $w^* < w^u$. And, when the profit potential in other markets from these wholesale price cuts is sufficiently large, the input market effect can outweigh the retail market effect for market i .

The next figure presents a graphical depiction of the underlying differences between *ad valorem* and unit taxes in the presence of a strategic supplier.

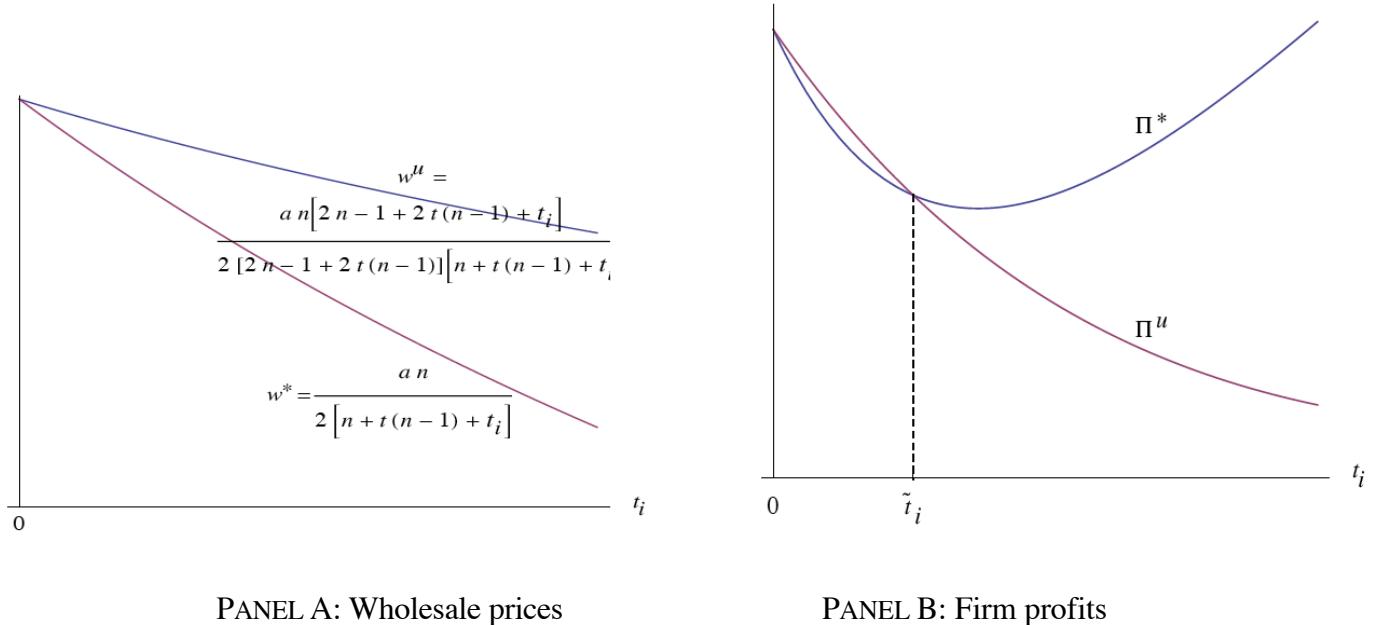


Figure 5: Unit Taxes vs. Ad Valorem Taxes.

3.5 Competition

The primary analysis centers around a simple supply chain setting of a monopolist supplier providing inputs to a monopolist retailer in order to highlight the key effects supply chains introduce to an examination of economic effects of sales taxes. We now append this primary analysis to consider a competitive retail market (section 3.5.1) and a competitive wholesale market (section 3.5.2).

3.5.1 COMPETITION IN THE OUTPUT MARKET

To examine if the results herein are robust to retail market competition and to consider the repercussions of competition in light of reliance on suppliers, consider the following extension – rather than the firm being a monopolist in each market, it now faces a (Cournot) competitor who also relies on its own (dedicated) supplier for inputs. Further, denote consumer demand in market i by $\hat{p}_i = a - q_i - \gamma q_{Ri}$ and $\hat{p}_{Ri} = a - q_{Ri} - \gamma q_i$, where \hat{p}_i (\hat{p}_{Ri}) represents the consumer out-of-pocket price for the firm's (rival's) good in market i , q_i (q_{Ri}) represents the retail quantity provided by the firm (rival) in market i , and γ , $\gamma \in [0,1]$, reflects the degree of competitive intensity (product substitutability) between the firm and rival.

In this setting, the relevant condition on consumer demand to ensure interior solutions is $a > \left[\frac{2(1+t^{\max})(1+\bar{t})}{2(1-t^{\max}) + \gamma t_i + (4-\gamma)\bar{t}} \right] [c + v]$. While the appendix derives the equilibrium under this modified scenario, the key conclusion, representing the analog to Proposition 3, is presented in the next Proposition.

PROPOSITION 10. Under output market competition,

(i) an increase in sales tax in market i increases the firm's profit for $t_i > t_i^*(\gamma)$, where

$t_i^*(\gamma)$ is the unique t_i -value that solves $t_i = f(\bar{t}; \gamma)$,

$$f(\bar{t}; \gamma) = [4 - \gamma] \left[\frac{(6 - \gamma)(2 - \gamma)}{(1 + \bar{t})^2} + \left(\frac{2[c + v]}{a} \right)^2 \right]^{-1/2} - 1.$$

(ii) the greater the degree of retail competition, the less often the firm benefits from an increase in sales taxes, i.e., $\frac{dt_i^*(\gamma)}{d\gamma} > 0$.

Proposition 10(i) confirms that the key conclusions of our primary analysis are robust in the presence of retail competition ($\gamma > 0$). In other words, it is the supply chain effects, not the presumed absence of retail competition, that drives our primary results. Proposition 10(ii) takes the next step to consider how the degree of competition alters this supply chain relationship. As retail competition increases, a firm's ability to parlay lower wholesale prices into greater retail profit are limited. As a result, the loss in retail demand from higher sales taxes becomes (relatively) more prominent, thereby reducing the circumstances under which firms actually benefit from hikes in tax rates in a market. This suggests that circumstances under which retailers are less likely to oppose proposals to increase (or introduce) sales taxes are those in which the retailer has substantial market power (concentration).

3.5.2 COMPETITION IN THE INPUT MARKET

Continuing the theme of competition, consider the case of nontrivial supply market competition. To examine varying degrees of supply competition most succinctly, consider the case of Cournot competition among N suppliers, $N \geq 1$ (see, e.g., Arya and Pfeiffer 2012). In this case, the condition for positive equilibrium retail quantities is

$$a > \left[\frac{N(1+t^{\max})(1+\bar{t})}{N-t^{\max}+(N+1)\bar{t}} \right] [c+v].$$

Relegating details to the appendix, the next proposition presents the analog to Proposition 3 in the case of supply market competition.

PROPOSITION 11. Under Cournot competition in the supply market,

(i) an increase in sales tax in market i increases the firm's profit for $t_i > t_i^*(N)$, where

$t_i^*(N)$ is the unique t_i -value that solves $t_i = f(\bar{t}; N)$,

$$f(\bar{t}; N) = \left[\frac{2N+1}{(N+1)^2(1+\bar{t})^2} + \left(\frac{N(c+v)}{(N+1)a} \right)^2 \right]^{-1/2} - 1.$$

(ii) the greater the degree of input market competition, the less often the firm benefits from an increase in sales taxes, i.e., $\frac{dt_i^*(N)}{dN} > 0$.

Proposition 11(i) generalizes the result of Proposition 3 to the case of supplier competition. The result confirms that it is not a monopolist supplier that matters, but rather imperfect coordination of the supply chain. Proposition 11(ii), however, confirms that greater competition does mitigate the supply market effects of sales taxes. As can be expected, for $N = 1$, $f(\bar{t}; N)$ reduces to the cutoff value in Proposition 3. As competition increases in the supply market, the extent to which sales taxes can cut wholesale prices is reduced since input market competition already serves to shrink the prevailing wholesale price. In other words, sales taxes and competition serve as substitutes when it comes to mitigating double marginalization along the supply chain. As for the limiting case of $N \rightarrow \infty$, this corresponds to the insourcing case since that entails marginal-cost pricing. In that limiting case, the condition in Proposition 11(i) reduces to $t_i > f(\bar{t}; N) = \frac{a}{c+v} - 1$, a condition that cannot be jointly satisfied with the nonnegativity condition – with perfect competition in the supply market, a retail firm can never benefit from a hike in sales tax rates. Beyond the limiting case, however, the condition for a preference for tax hikes is non-trivial, i.e., for all finite N , there always exist parameters that satisfy the condition $t_i > f(\bar{t}; N)$.

The next figure provides a pictorial summary of the effects of competition, both in the retail and supply realms.

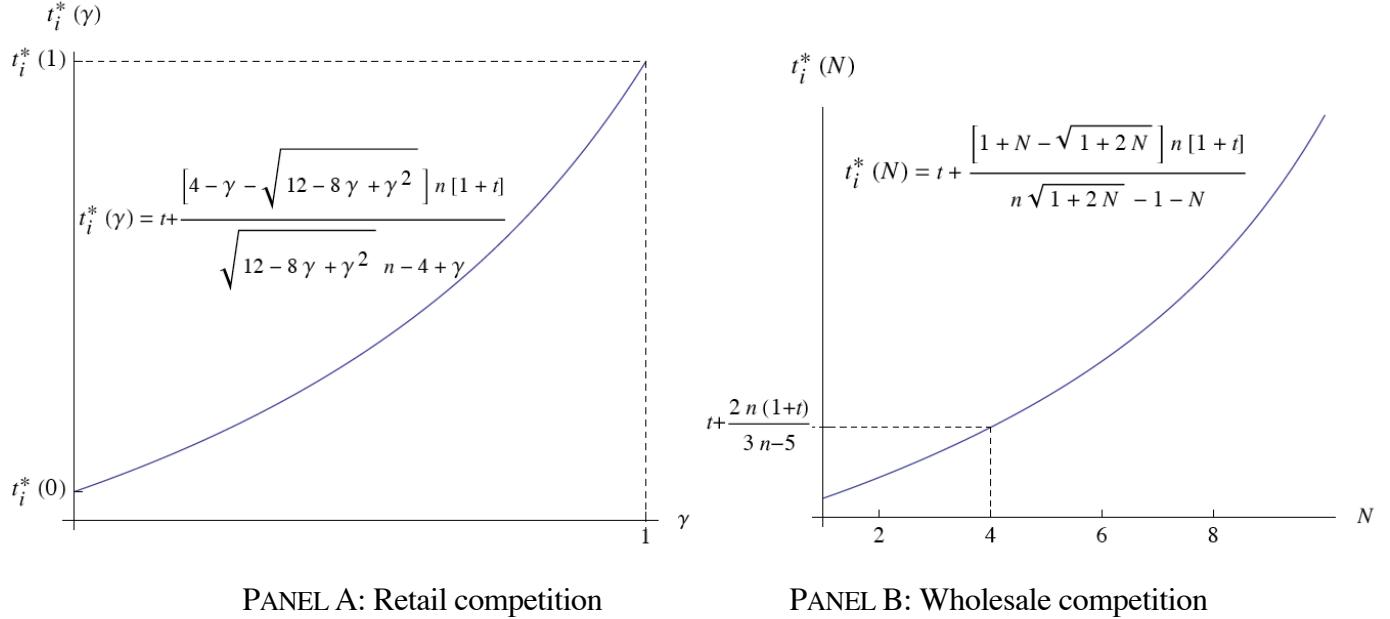


Figure 6: Effects of Competition on Preference for Sales Tax Increase.

4. Implications and Conclusion

While it has long been recognized that consumer-demand effects of retail sales taxes can present notable economic repercussions, little if any attention has been paid to supply chain consequences of such taxes. This paper examines such consequences and finds that sales taxes can have notable effects on supply markets and these effects alter traditional views of tax policy when (i) suppliers exhibit pricing power and (ii) retail firms operate in multiple tax jurisdictions.

In particular, we show that increases in sales taxes in one retail market not only undercut retail demand but also wholesale demand (this despite the fact that wholesale purchases themselves are not taxed). This reduced wholesale demand compels the supplier to cut its input price to maintain demand for its goods in the high-tax market. This lower

input price, in turn, permits a retailer to provide more and cheaper goods to consumers in other markets.

The results introduce caveats to the traditional views of sales taxes and offer some implications for future study. In terms of the former, not only do the results conclude that the most basic premise of conventional wisdom – that consumers and firms prefer taxes to be lower – may not always be true, they also suggest that tax jurisdictions may be able to leverage this to gain support for unilateral shifts in taxes. A key driver of this result arises when markets are characterized by potential differences in tax rates, as is the case across states and across sales platforms (in-store vs. online). In that case, attempts to raise tax rates in one market may actually face limited resistance (and even tacit support) by retailers due to the spillover effects to the retailer's other markets. Among other things, this may reflect why retailers have fought aggressively to limit collecting sales/use taxes for online purchases but show less aggression in fighting local sales tax hikes.

The results also demonstrate that due to supplier pricing effects, welfare is maximized when tax burdens are shared equally across markets. This suggests that well-intentioned efforts to target tax breaks to some product or consumer groups may have deleterious effects because the supply pricing consequences will lead to a counterproductive shift of resources to particular markets or consumers.

Finally, we note that our results provide some empirical implications for the well-established streams of literature on tax incidence. In particular, the fundamental question of whether a tax burden levied at the consumer level will fully be borne by consumers or whether further retail price hikes will lead the tax to be "overshifted" has been extensively studied, with mixed empirical evidence. Our results suggest a mitigating factor to overshifting is the degree to which supply markets are coordinated. A fully coordinated or highly competitive supply market will favor the overshifting often implied by existing models. However, our results show that when supply markets exhibit powerful and strategic suppliers, the imposition of additional tax on consumers may support the notion of

"undershifting" thanks to input price cuts; this tension is consistent with the partial tax shifting observed empirically in some (but not all) retail markets.

Though the focus here is on taxes levied on purchases in particular retail markets, the results are also suggestive of effects of government subsidies tied to purchases (negative sales taxes) seen in certain markets. For example, when tax credits are tied to purchases of specific energy-efficient products, the conventional wisdom is that the boost in demand will promote energy efficiency. The results here note that the subsidy can also lead to a concomitant hike in supplier prices and that this price hike may actually raise retail prices of other products using energy efficient technologies that are not subject to tax incentives. Thus, while targeted subsidies may boost sales of the energy efficient products targeted, they can also have the unintended consequence of undercutting sales of other energy efficient product categories.

Taken one step further, the results also suggest an additional avenue of study: when government incentives seek to boost demand for socially-beneficial services (e.g., higher education or medical care), a full understanding of the policy consequences would require consideration of how strategic input suppliers (in particular, labor), respond to such consumer incentives. Future study could examine these broader consequences in a model of labor supply.

APPENDIX

Proof of Proposition 1. Under insourcing, the firm's production decision is obtained by solving the following problem:

$$\underset{q_1, \dots, q_n}{\text{Max}} \quad \sum_{i=1}^n \left[\frac{a - q_i}{1 + t_i} - c - v \right] q_i. \quad (\text{A1})$$

The first-order condition of (A1) yields $q_i(v) = [1/2] \left[a - (1 + t_i)(c + v) \right]$. The non-negativity condition noted in the text implies $a > [1 + t_i][c + v]$ ensuring $q_i(v) > 0$. Setting $w = v$ in (2), (3), and (4) yields tax collections, firm profits, and welfare expressions in the insourcing case. Taking appropriate derivatives then yields:

$$\begin{aligned} \frac{d\Pi(v)}{dt_i} &= -\frac{a^2 - [1 + t_i]^2 [c + v]^2}{4[1 + t_i]^2}; \\ \frac{dT_i(v)}{dt_i} &= \frac{a^2 - [1 + t_i]^2 [1 + 2t_i][c + v]^2}{4[1 + t_i]^2} \text{ and } \frac{dT_j(v)}{dt_i} = 0, \quad j \neq i; \\ \frac{d\hat{p}_i(v)}{dt_i} &= \frac{d[a - q_i(v)]}{dt_i} = \frac{c + v}{2} \text{ and } \frac{d\hat{p}_j(v)}{dt_i} = \frac{d[a - q_j(v)]}{dt_i} = 0; \text{ and} \\ \frac{d\Psi(v)}{dt_i} &= \frac{d\left\{ [a - q_i(v) - c - v]q_i(v) + q_i^2(v)/2 \right\}}{dt_i} = -\frac{[a - (1 + t_i)(c + v)][c + v]}{4}. \end{aligned}$$

Given $a > [1 + t_i][c + v]$, $\frac{d\Pi(v)}{dt_i} < 0$, $\frac{d\hat{p}_i(v)}{dt_i} > 0$, and $\frac{d\Psi(v)}{dt_i} < 0$. The comparative static on $T_i(v)$ is positive if $\frac{a}{[1 + t_i][c + v]} > \sqrt{1 + 2t_i}$, and negative otherwise.

Proof of Lemma 1. Under outsourcing, solving (A1) with v replaced by w yields $q_i(w) = [1/2] \left[a - (1 + t_i)(c + w) \right]$. Thus, the supplier's problem is:

$$\underset{w}{\text{Max}} \quad [w - v] \sum_{i=1}^n q_i(w) \equiv \frac{n}{2} [w - v] [a - (1 + \bar{t})(c + w)]. \quad (\text{A2})$$

The first-order condition of (A2) yields w^* in Lemma 1, and $q_i^* = q_i(w^*)$. Finally, $\Pi^* = \sum_{i=1}^n \left[\frac{a - q_i^*}{1 + t_i} - c - w^* \right] q_i^*$ and $T_i^* = t_i \left[\frac{a - q_i^*}{1 + t_i} \right] q_i^*$. The lower bound on a noted in text corresponds to $q_i^* > 0$ for all i .

Proof of Proposition 2. Using w^* from Lemma 1,

$$\frac{dw^*}{dt_i} = -\frac{a}{2[1+\bar{t}]^2} \frac{d\bar{t}}{dt_i} = -\frac{a}{2n[1+\bar{t}]^2} < 0.$$

Proof of Proposition 3. Using Π^* from Lemma 1,

$$\begin{aligned} \frac{d\Pi^*}{dt_i} &= \frac{1}{16} \left[a^2 \left(-\frac{4}{[1+t_i]^2} + \frac{3}{[1+\bar{t}]^2} \right) + (c+v)^2 \right] \text{ and} \\ \frac{d^2\Pi^*}{dt_i^2} &= \frac{a^2}{8} \left[\frac{4n[1+\bar{t}]^3 - 3[1+t_i]^3}{n[1+t_i]^3[1+\bar{t}]^3} \right]. \end{aligned} \quad (\text{A3})$$

Note $4n[1+\bar{t}]^3 - 3[1+t_i]^3 > 4n[1+t_i/n]^3 - 3[1+t_i]^3 > 3$; the last inequality follows from the fact that the minimum value of $4n[1+t_i/n]^3 - 3[1+t_i]^3$ for $n \geq 2$ and $0 \leq t_i \leq 1$ is 3 which corresponds to the choice of $n = 2$ and $t_i = 1$. Thus, from the second equality in (A3), Π^* is convex in t_i , i.e., $d^2\Pi^*/dt_i^2 > 0$. Given this, $d\Pi^*/dt_i > 0$ for all $t_i > t_i^*$, where t_i^* is the unique solution that solves $d\Pi^*/dt_i = 0$. From the first equality in (A3), $d\Pi^*/dt_i = 0$ is equivalent to the condition $t_i = f(\bar{t})$, with $f(\bar{t})$ as noted in Proposition 3.

Proof of Proposition 4. Using Π^* from Lemma 1 and, replacing t_i by $t_i + \Delta$, $\Delta \geq 0$, for all i , yields $\Pi^*(\Delta)$, the firm's profit as a function of Δ :

$$\Pi^*(\Delta) = \frac{1}{4} \left[a^2 \sum_{i=1}^n \left(\frac{1}{1+t_i + \Delta} \right) - \frac{n}{4} \left(\frac{3a^2}{1+\bar{t}+\Delta} + [c+v][2a - (1+\bar{t}+\Delta)(c+v)] \right) \right].$$

Taking the derivative of the above expression with respect to Δ yields:

$$\frac{d\Pi^*(\Delta)}{d\Delta} = -\frac{a^2}{4} \left[\left\{ \sum_{i=1}^n \left(\frac{1}{1+t_i + \Delta} \right)^2 - \frac{n}{(1+\bar{t}+\Delta)^2} \right\} + \frac{n}{4} \left\{ \frac{1}{(1+\bar{t}+\Delta)^2} - \left(\frac{c+v}{a} \right)^2 \right\} \right].$$

The proof follows from the fact that each term in the $\{ \}$ -brackets is positive as we next show. The second term in the $\{ \}$ -brackets is positive since the non-negativity condition on quantities implies $a > [1+\bar{t}+\Delta][c+v]$. The firm term in the $\{ \}$ -brackets is positive if $(1+\bar{t}+\Delta)^2 > \frac{n}{\sum_{i=1}^n \left(\frac{1}{1+t_i + \Delta} \right)^2}$. This is proved below making use of the facts

that: (i) the arithmetic mean is greater than the harmonic mean for positive data, leading to the first inequality and (ii) $E[x_i^2] > E^2[x_i]$, where $x_i = 1/[1+t_i + \Delta]$, giving the second inequality:

$$(1 + \bar{t} + \Delta)^2 > \frac{n^2}{\left(\sum_{i=1}^n \frac{1}{1 + t_i + \Delta} \right)^2} > \frac{n^2}{n \sum_{i=1}^n \left(\frac{1}{1 + t_i + \Delta} \right)^2} = \frac{n}{\sum_{i=1}^n \left(\frac{1}{1 + t_i + \Delta} \right)^2}.$$

Proof of Proposition 5. Using T_j^* , $j \neq i$, in (6),

$$\frac{dT_j^*}{dt_i} = \frac{at_j[1 + t_j][a + (1 + t_j)(c + v)]}{8n[1 + \bar{t}]^3} > 0.$$

Using T_i^* in (6),

$$\begin{aligned} \frac{d^2T_i^*}{dt_i^2} &= -\frac{[c + v] \left[2a \left\{ t_i(1 + t_i) - n(1 + 2t_i)(1 + \bar{t}) + n^2(1 + \bar{t})^2 \right\} + (c + v)n^2(1 + \bar{t})^3 \right]}{8n^2[1 + \bar{t}]^3} - \\ &\quad \frac{a^2 \left[3t_i(1 + t_i)^4 - 2n(1 + t_i)^3(1 + 2t_i)(1 + \bar{t}) + n^2(1 + \bar{t})^2(5 + 3t_i + 3t_i^2 + t_i^3 + 8\bar{t} + 4\bar{t}^2) \right]}{8n^2[1 + t_i]^3[1 + \bar{t}]^4}. \end{aligned}$$

For $0 < t_i < 1$ and $n \geq 2$, the terms $t_i(1 + t_i) - n(1 + 2t_i)(1 + \bar{t}) + n^2(1 + \bar{t})^2 > 0$ and $3t_i(1 + t_i)^4 - 2n(1 + t_i)^3(1 + 2t_i)(1 + \bar{t}) + n^2(1 + \bar{t})^2(5 + 3t_i + 3t_i^2 + t_i^3 + 8\bar{t} + 4\bar{t}^2) > 0$. Thus, it follows that T_i^* is concave in t_i , i.e., $d^2T_i^* / dt_i^2 < 0$. Given concavity, $dT_i^* / dt_i > 0$ for $t_i < t_i^T$ where t_i^T is the t_i -value that solves $dT_i^* / dt_i = 0$. From T_i^* in (6), $\frac{dT_i^*}{dt_i} = \frac{g(t_i, \bar{t})}{16n[1 + t_i]^2[1 + \bar{t}]^3}$, where $g(t_i, \bar{t})$ is noted in Proposition 5. Thus, t_i^T is the unique t_i -value that solves $g(t_i, \bar{t}) = 0$.

Proof of Proposition 6. Using Lemma 1,

$$\frac{d\hat{p}_i^*}{dt_i} = \frac{d[a - q_i^*]}{dt_i} = \frac{a[n - 1 + n\bar{t} - t_i]}{4n[1 + \bar{t}]^2} + \frac{c + v}{4} > 0 \text{ and}$$

$$\frac{d\hat{p}_j^*}{dt_i} = \frac{d[a - q_j^*]}{dt_i} = -\frac{a[1 + t_j]}{4n[1 + \bar{t}]^2} < 0.$$

Proof of Proposition 7. Since $\Psi^* = \sum_{i=1}^n \left([a - q_i^*]q_i^* - vq_i^* - cq_i^* + \frac{[q_i^*]^2}{2} \right)$, from

Lemma 1,

$$\begin{aligned} \Psi^* &= \frac{na^2[7 + 14\bar{t} + 7\bar{t}^2 - \sigma_t^2]}{32[1 + \bar{t}]^2} - \\ &\quad \frac{n[c + v] \left[2a \left\{ 7 + 10\bar{t} + 3\bar{t}^2 + \sigma_t^2 \right\} - (c + v)(1 + \bar{t}) \left\{ 7 + 6\bar{t} - \bar{t}^2 - \sigma_t^2 \right\} \right]}{32[1 + \bar{t}]} \end{aligned}$$

Using the above,

$$\frac{d^2\Psi^*}{dt_i^2} = -\frac{a^2[n(1+\bar{t})^2 + 2\bar{t} + 3\bar{t}^2 - 1 - 4t_i(1+\bar{t}) + 3\sigma_t^2]}{16n[1+\bar{t}]^4} - \frac{[c+v]\left[2a\left\{n(1+\bar{t})^2 + \bar{t}^2 - 1 - 2t_i(1+\bar{t}) + \sigma_t^2\right\} + n(c+v)(1+\bar{t})^3\right]}{16n[1+\bar{t}]^3} < 0.$$

The above inequality follows from the fact that, given the lower bound on a (i.e., the non-negativity condition on quantities), $\bar{t} > t_i / n$, $\sigma_t^2 > [t_i - \bar{t}]^2 / n$, and $n \geq 2$, the numerator of each of the two terms on the right-hand-side of $d^2\Psi^* / dt_i^2$ is positive, i.e.,

$$n(1+\bar{t})^2 + 2\bar{t} + 3\bar{t}^2 - 1 - 4t_i(1+\bar{t}) + 3\sigma_t^2 > 0 \quad \text{and}$$

$$2a\left\{n(1+\bar{t})^2 + \bar{t}^2 - 1 - 2t_i(1+\bar{t}) + \sigma_t^2\right\} + n(c+v)(1+\bar{t})^3 > 0.$$

Given concavity, $d\Psi^* / dt_i > 0$ for $t_i < t_i^\Psi$ where t_i^Ψ is the t_i -value that solves $d\Psi^* / dt_i = 0$. From Ψ^* above, $\frac{d\Psi^*}{dt_i} = \frac{h(t_i, \bar{t}, \sigma_t^2)}{16[1+\bar{t}]^3}$, where $h(t_i, \bar{t}, \sigma_t^2)$ is noted in Proposition 7. Thus, t_i^Ψ is the unique t_i -value that solves $h(t_i, \bar{t}, \sigma_t^2) = 0$.

Proof of Lemma 2. Under unit taxes, the firm chooses quantities by solving the following problem:

$$\underset{q_1, \dots, q_n}{\text{Max}} \quad \sum_{i=1}^n [a - q_i - c - w - t_i'] q_i.$$

The first-order condition of the above yields $q_i(w) = [1/2] [a - c - t_i' - w]$. Thus, the supplier's problem is:

$$\underset{w}{\text{Max}} \quad [w - v] \sum_{i=1}^n q_i(w) \equiv \frac{n}{2} [w - v] [a - c - \bar{t}' - w].$$

The solution to the supplier's problem yields w' ; and, so, $q_i' = q_i(w')$; under this solution the firm's profit equals Π' .

Proof of Proposition 8. Using the firm profit expressions from Lemma 2, the results follows from the following comparative statics:

$$\frac{d\Pi'}{dt_i'} = \frac{1}{8} [4t_i' - a - 3\bar{t}' + c + v] \quad \text{and} \quad \frac{d\Pi''}{dt_i''} = -\frac{1}{16} [a - c - v]^2 < 0.$$

Proof of Proposition 9. Consider the case of insourcing. Lemma 1 provides the quantities under sales tax. With $w = v$, the proof of Lemma 2 provides the quantities under

unit tax. For the consumers in market i to be indifferent, the consumer price (and, hence, the quantity) in market i should be equal, i.e.,

$$\frac{a - [1 + t_i][c + v]}{2} = \frac{a - t_i^u - c - v}{2} \Rightarrow t_i^u = t_i[c + v].$$

The firm's profit under sales tax is $\Pi(v)$; with the shift to unit tax in market i and $t_i^u = t_i[c + v]$, the firm's profit is $\Pi^u(v)$, where:

$$\begin{aligned} \Pi(v) &= \sum_{j=1}^n \frac{[a - (1 + t_j)(c + v)]^2}{4[1 + t_j]} \text{ and} \\ \Pi^u(v) &= \frac{[a - (1 + t_i)(c + v)]^2}{4} + \sum_{\substack{j=1 \\ j \neq i}}^n \frac{[a - (1 + t_j)(c + v)]^2}{4[1 + t_j]}. \end{aligned}$$

Thus, $\Pi^u(v) - \Pi(v) = \frac{t_i[a - (1 + t_i)(c + v)]^2}{4[1 + t_i]} > 0$, proving part (i). Under outsourcing, with the shift to unit tax in market i , the firm's problem is:

$$\underset{q_1, \dots, q_n}{\text{Max}} \left[a - q_i - c - w - t_i^u \right] q_i + \sum_{\substack{j=1 \\ j \neq i}}^n \left[\frac{a - q_j}{1 + t_j} - c - w \right] q_j. \quad (\text{A4})$$

The first-order condition of (A4) yields:

$$q_i(w) = [1/2][a - c - t_i^u - w] \text{ and } q_j(w) = [1/2][a - (1 + t_j)(c + w)]. \quad (\text{A5})$$

Using (A5), the supplier's problem is:

$$\underset{w}{\text{Max}} \left[w - v \right] [q_i(w) + \sum_{\substack{j=1 \\ j \neq i}}^n q_j(w)] \equiv \frac{1}{2} [w - v] \left[a - t_i^u - c - w + [n - 1][a - (1 + \bar{t})(c + w)] \right].$$

The above problem yields:

$$w^u = \frac{1}{2} \left[\frac{an - t_i^u}{n(1 + \bar{t}) - t_i} - c + v \right]. \quad (\text{A6})$$

For the consumers in market i to be indifferent, the quantity in market i should be equal under the unit tax and the sales tax, i.e.,

$$q_i(w^u) = q_i^* \Leftrightarrow [1/2][a - c - t_i^u - w^u] = \frac{a}{2} - \frac{[1 + t_i][a + (1 + \bar{t})(c + v)]}{4[1 + \bar{t}]}.$$

Solving the above for t_i^u yields:

$$t_i^u = \left[\frac{t_i}{2n(1+\bar{t}) - 2t_i - 1} \right] \left[\frac{a[n(1+\bar{t}) - 1 - t_i]}{1+\bar{t}} + [n(1+\bar{t}) - t_i][c + v] \right]. \quad (\text{A7})$$

Using quantities from (A5), wholesale price from (A6), and the unit tax rate from (A7) in (A4) yields firm profit of:

$$\Pi^u = \frac{1}{4} \left[a - t_i^u - c - w^u \right]^2 + \frac{1}{4} \left[a^2 \sum_{\substack{j=1 \\ j \neq i}}^n \left(\frac{1}{1+t_j} \right) - 2a[n-1][c + w^u] + [n(1+\bar{t}) - 1 - t_i][c + w^u]^2 \right].$$

Using Π^u from above and Π^* from Lemma 1,

$$\Pi^* - \Pi^u \Big|_{c=v=0} = \frac{a^2 t_i [At_i^2 + Bt_i + C]}{16[1+t_i][n(1+\bar{t}) + t_i - \bar{t}]^2[1+2\bar{t} - 2n(1+\bar{t})]^2}, \text{ where} \quad (\text{A8})$$

$$A = -4n^4[1+\hat{t}]^2 - 4[1+2\hat{t}]^2 + 24n[1+3\hat{t} + 2\hat{t}^2] + 3n^3[9 + 17\hat{t} + 8\hat{t}^2] - n^2[44 + 99\hat{t} + 52\hat{t}^2];$$

$$B = 8\hat{t}[1+2\hat{t}]^2 + 4n^4[1+\hat{t}]^2[3 + 4\hat{t}] - 56n\hat{t}[1+3\hat{t} + 2\hat{t}^2] - n^3[23 + 135\hat{t} + 192\hat{t}^2 + 80\hat{t}^3] + n^2[9 + 143\hat{t} + 284\hat{t}^2 + 144\hat{t}^3]; \text{ and}$$

$$C = -16n^4\hat{t}[1+\hat{t}]^3 - 4\hat{t}^2[1+2\hat{t}]^2 + 32n\hat{t}^2[1+3\hat{t} + 2\hat{t}^2] + 2n^3[-1 + 11\hat{t} + 60\hat{t}^2 + 80\hat{t}^3 + 32\hat{t}^4] - n^2[-1 + 6\hat{t} + 100\hat{t}^2 + 192\hat{t}^3 + 96\hat{t}^4].$$

From (A8), $\Pi^* - \Pi^u \Big|_{c=v=0} > 0$ if and only if $At_i^2 + Bt_i + C > 0$. At $t_i = 0$, $At_i^2 + Bt_i + C = C < 0$. Also, $\frac{d[At_i^2 + Bt_i + C]}{dt_i} = 2At_i + B > 0$. Thus, the \tilde{t}_i cutoff corresponds to the t_i -value in $[0,1]$ that solves the quadratic $At_i^2 + Bt_i + C = 0$. This root is noted in part (ii).

Proof of Proposition 10. Under (Cournot) retail competition, the firm and the rival solve the following problems, respectively:

$$\underset{q_1, \dots, q_n}{\text{Max}} \sum_{i=1}^n \left[\frac{a - q_i - \gamma q_{Ri}}{1+t_i} - c - w \right] q_i \text{ and } \underset{q_{R1}, \dots, q_{Rn}}{\text{Max}} \sum_{i=1}^n \left[\frac{a - q_{Ri} - \gamma q_i}{1+t_i} - c - w_R \right] q_{Ri}. \quad (\text{A9})$$

Simultaneously solving the first-order conditions of the above problems yields:

$$q_i(w, w_R; \gamma) = \frac{1}{2+\gamma} \left[a - (1+t_i)(c + \frac{2w - \gamma w_R}{2-\gamma}) \right] \text{ and}$$

$$q_{Ri}(w, w_R; \gamma) = \frac{1}{2+\gamma} \left[a - (1+t_i)(c + \frac{2w_R - \gamma w}{2-\gamma}) \right]. \quad (\text{A10})$$

Given (A10), the firm's supplier chooses w to maximize $\sum_{i=1}^n [w - v] q_i(w, w_R; \gamma)$, while the rival's supplier sets w_R to maximize $\sum_{i=1}^n [w - v] q_{Ri}(w, w_R; \gamma)$. Jointly solving the two first-order conditions yields the wholesale prices $w^*(\gamma)$ and $w_R^*(\gamma)$; using these in (A10) yields the quantities $q_i^*(\gamma)$ and $q_{Ri}^*(\gamma)$; and using these prices and quantities in (A9) gives firm profit of $\Pi^*(\gamma)$. These expressions are presented below:

$$\begin{aligned} w^*(\gamma) &= w_R^*(\gamma) = \frac{1}{4-\gamma} \left[\frac{a[2-\gamma]}{1+\bar{t}} + 2v - c[2-\gamma] \right]; \\ q_i^*(\gamma) &= q_{Ri}^*(\gamma) = \frac{1}{[4-\gamma][2+\gamma]} \left[\frac{a[2(1-t_i) + \gamma t_i + (4-\gamma)\bar{t}]}{1+\bar{t}} - 2[1+t_i][c+v] \right]; \text{ and} \\ \Pi^*(\gamma) &= \frac{1}{[2+\gamma]^2} \left[a^2 \sum_{i=1}^n \left(\frac{1}{1+t_i} \right) - \frac{n}{[4-\gamma]^2} \left(\frac{a^2[6-\gamma][2-\gamma]}{1+\bar{t}} + 4[c+v][2a - (1+\bar{t})(c+v)] \right) \right]. \end{aligned}$$

From the expression for quantities above, the non-negativity condition in the retail competition setting is $a > \left[\frac{2(1+t^{\max})(1+\bar{t})}{2(1-t^{\max}) + \gamma t_i + (4-\gamma)\bar{t}} \right] [c+v]$. Also, from the above,

$$\frac{d\Pi^*(\gamma)}{dt_i} = \frac{1}{[2+\gamma]^2} \left[a^2 \left(-\frac{1}{[1+t_i]^2} + \frac{[6-\gamma][2-\gamma]}{[4-\gamma]^2[1+\bar{t}]^2} \right) + \frac{4(c+v)^2}{[4-\gamma]^2} \right]. \quad (\text{A11})$$

Some tedious calculation confirm that $\frac{d^2\Pi^*(\gamma)}{dt_i^2} > 0$. Thus, for $t_i > t_i^*(\gamma)$, $\frac{d\Pi^*(\gamma)}{dt_i} > 0$, where $t_i^*(\gamma)$ is the t_i -value that solves $\frac{d\Pi^*(\gamma)}{dt_i} = 0$. From (A11), $\frac{d\Pi^*(\gamma)}{dt_i} = 0$ is equivalent to the condition $t_i = f(\bar{t}; \gamma)$, where $f(\bar{t}; \gamma)$ is as in part (i). Turning to part (ii),

$$\begin{aligned} \frac{dt_i^*(\gamma)}{d\gamma} &= \frac{\partial f(\bar{t}; \gamma)}{\partial \bar{t}} \frac{\partial \bar{t}}{\partial \gamma} + \frac{\partial f(\bar{t}; \gamma)}{\partial \gamma} = \frac{\partial f(\bar{t}; \gamma)}{\partial \bar{t}} \frac{1}{n} \frac{dt_i^*(\gamma)}{d\gamma} + \frac{\partial f(\bar{t}; \gamma)}{\partial \gamma} \\ &\Rightarrow \frac{dt_i^*(\gamma)}{d\gamma} \left[1 - \frac{1}{n} \frac{\partial f(\bar{t}; \gamma)}{\partial \bar{t}} \right] = \frac{\partial f(\bar{t}; \gamma)}{\partial \gamma}. \end{aligned} \quad (\text{A12})$$

Using $f(\bar{t}; \gamma)$ from part (i),

$$\frac{\partial f(\bar{t}; \gamma)}{\partial \gamma} = \frac{4a[1+\bar{t}]}{\left[a^2(12-8k+k^2) + 4(1+\bar{t})^2(c+v)^2 \right]^{3/2}} \left[a^2 - (1+\bar{t})^2(c+v)^2 \right] > 0 \text{ and}$$

$$\frac{1}{n} \frac{\partial f(\bar{t}; \gamma)}{\partial \bar{t}} = \frac{[6 - \gamma][2 - \gamma][1 + f(\bar{t}; \gamma)]^3}{n[1 + \bar{t}]^3[4 - \gamma]^2} < 1. \quad (\text{A13})$$

From (A12) and (A13), $\frac{dt_i^*(\gamma)}{d\gamma} > 0$.

Proof of Proposition 11. As noted in the proof of Lemma 1, $q_i(w) = [1/2] \left[a - (1 + t_i)(c + w) \right]$. Let y_j , $j = 1, \dots, N$, denote the quantity supplied by the j th supplier. Equating total demand with total supply,

$$\sum_{i=1}^n q_i(w) = \sum_{j=1}^N y_j \Rightarrow w = \frac{an - cn[1 + \bar{t}] - 2 \sum_{j=1}^N y_j}{n[1 + \bar{t}]} \quad (\text{A14})$$

Given (A14), and taking supply of other suppliers as given under (Cournot) input market competition, supplier k solves:

$$\underset{y_k}{\text{Max}} \left[\frac{an - cn[1 + \bar{t}] - 2 \sum_{\substack{j=1 \\ j \neq k}}^N y_j - 2y_k}{n[1 + \bar{t}]} - v \right] y_k, \quad k = 1, \dots, N. \quad (\text{A15})$$

Jointly solving the first-order conditions of the problem in (A15) yields $y_k^*(N)$; using this in (A14) yields $w^*(N)$; $q_i(w^*(N))$ is then denoted $q_i^*(N)$; and firm profit under these choices of quantity and input price is denoted $\Pi^*(N)$:

$$y_j^*(N) = \frac{n[a - (1 + \bar{t})(c + v)]}{2[N + 1]}, \quad w^*(N) = \frac{1}{N + 1} \left[\frac{a}{1 + \bar{t}} - c + Nv \right];$$

$$q_i^*(N) = \frac{1}{2(N + 1)} \left[\frac{a[N - t_i + (N + 1)\bar{t}]}{1 + \bar{t}} - N(1 + t_i)(c + v) \right]; \text{ and}$$

$$\Pi^*(N) = \sum_{i=1}^n \left\{ \frac{[a(N - t_i + [N + 1]\bar{t}) - N(1 + t_i)(1 + \bar{t})(c + v)]^2}{4[N + 1]^2[1 + \bar{t}]^2[1 + t_i]} \right\}.$$

From the expression for quantities above, the non-negativity condition in the supplier competition setting is $a > \left[\frac{N(1 + t^{\max})(1 + \bar{t})}{N - t^{\max} + (N + 1)\bar{t}} \right] [c + v]$. Also, from the above,

$$\frac{d\Pi^*(N)}{dt_i} = \frac{1}{4} \left[a^2 \left(-\frac{1}{[1+t_i]^2} + \frac{2N+1}{[N+1]^2[1+\bar{t}]^2} \right) + \frac{N^2(c+v)^2}{[N+1]^2} \right]. \quad (\text{A16})$$

It can be easily confirmed that $\frac{d^2\Pi^*(N)}{dt_i^2} > 0$. Thus, for $t_i > t_i^*(N)$, $\frac{d\Pi^*(N)}{dt_i} > 0$, where $t_i^*(N)$ is the t_i -value that solves $\frac{d\Pi^*(N)}{dt_i} = 0$. From (A16), $\frac{d\Pi^*(N)}{dt_i} = 0$ is equivalent to the condition $t_i = f(\bar{t}; N)$, where $f(\bar{t}; N)$ is as in part (i). Turning to part (ii),

$$\begin{aligned} \frac{dt_i^*(N)}{dN} &= \frac{\partial f(\bar{t}; N)}{\partial \bar{t}} \frac{\partial \bar{t}}{\partial N} + \frac{\partial f(\bar{t}; N)}{\partial N} = \frac{\partial f(\bar{t}; N)}{\partial \bar{t}} \frac{1}{n} \frac{dt_i^*(N)}{dN} + \frac{\partial f(\bar{t}; N)}{\partial N} \\ &\Rightarrow \frac{dt_i^*(N)}{dN} \left[1 - \frac{1}{n} \frac{\partial f(\bar{t}; N)}{\partial \bar{t}} \right] = \frac{\partial f(\bar{t}; N)}{\partial N}. \end{aligned} \quad (\text{A17})$$

Using $f(\bar{t}; N)$ from part (i),

$$\begin{aligned} \frac{\partial f(\bar{t}; N)}{\partial N} &= \frac{Na[1+\bar{t}]\left[a^2 - (1+\bar{t})^2(c+v)^2\right]}{\left[a^2(2N+1) + N^2(1+\bar{t})^2(c+v)^2\right]^{3/2}} > 0 \text{ and} \\ \frac{1}{n} \frac{\partial f(\bar{t}; N)}{\partial \bar{t}} &= \frac{[2N+1][1+f(\bar{t}; N)]^3}{n[1+\bar{t}]^3[N+1]^2} < 1. \end{aligned} \quad (\text{A18})$$

From (A17) and (A18), $\frac{dt_i^*(N)}{dN} > 0$.

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