

## *Factor Tariffs and Income*

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A change in the price of an imported primary factor of production lowers and rearranges output and redistributes income. Consider a factor tariff in a competitive small open economy producing two traded goods combining imported energy with domestic capital and labor. Suppose export production is energy intensive, and import competing production labor intensive. A tariff shifts production toward the import competing good, raises the wage, and lowers the capital return. The present paper shows that under some conditions the decreased import spending can outweigh the decreased value of output.

Keywords: Factor tariffs, income, general equilibrium

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## *Factor Tariffs and Income*

A tariff on an imported factor of production lowering the import shrinks the production frontier as outputs and domestic factor prices adjust. Consider an economy producing two goods under constant returns with two domestic factors of production and an imported factor. Assume the small open economy is a price taker in international markets for the two traded goods and the imported factor.

Assume imported energy is combined with domestic capital and labor to produce an export and an import competing good. Energy is the most intensive or extreme factor in export production, and labor in import competing production. An energy tariff lowers import, shifts production toward the import competing good, and raises the wage. The return to capital falls as it is released from export production. These effects are strengthened if energy and capital are technical complements, a possibility noted in the production literature.

The present paper shows that the decrease in energy import spending may outweigh the decrease in output. The potential increase in income depends on substitution and factor intensity between the three inputs as well as the state of the economy. This result is a characteristic of models with many goods and many factors of production, applying to imports of capital or natural resources as well as energy.

The first section reviews the fundamental production model of Mundell (1957) with an international factor of production available at an exogenous world price. The second section presents the present comparative static model of Thompson (1983) followed by a section focused on adjustments to a factor tariff.

## 1. A factor tariff, output, and income

The literature on an internationally mobile factor of production in general equilibrium models of production for a small open economy includes Kemp (1966), Jones (1967), Chipman (1971), Caves (1971), Jones and Ruffin (1975), Ferguson (1978), Srinivasan (1983), Svensson (1984), and Ethier and Svensson (1986). An increase in the exogenous price of an international factor lowers its import and leads to adjustments in production and domestic factor prices.

In a related model, Ruffin (1969) considers an imported intermediate good entering production in fixed proportions. Panagaria (1992) finds a tariff on the intermediate good has an ambiguous effect on utility. The present model finds an analogous ambiguous effect on income based on substitution between the imported factor and two domestic factors of production.

Adjustments to a tariff are pictured with the production frontier in Figure 1. Endowments of domestic factors and the level of the imported factor along with the two production functions determine position of the production frontier. The economy produces at point P given the terms of trade  $t$  and the price  $w_1$  of the imported factor. Export of good 1 must at least cover factor import spending  $w_1v_1$  where  $v_1$  is the import level. Real income in terms of good 1 is determined at the intercept  $y_0$  on the  $x_1$  axis.

\* Figure 1 \*

Assume the imported factor is energy and export production is energy intensive. An energy tariff reduces import  $v_1$  shrinking the production frontier and lowering export production  $x_1$ . Import competing output  $x_2$  may increase as in Figure 1 but both outputs may fall. Increased real income is illustrated at  $y_1$  as import spending  $w_1v_1$  falls more than the value of output.

Payments to the three factors exhaust output,

$$x \equiv \sum_j p_j x_j = (1 + t)w_1v_1 + w_2v_2 + w_3v_3, \quad (1)$$

where

$x \equiv$  output

$x_j \equiv$  output of good  $j$

$p_j \equiv$  price of good  $j$

$w_i \equiv$  price of factor  $i$

$v_1 \equiv$  import of factor 1

$v_2, v_3 \equiv$  domestic factors endowments

$t \equiv$  tariff rate.

Income is the value of output less import spending, equivalent to the sum of tariff revenue and

domestic factor payments,

$$y = x - w_1 v_1 = t w_1 v_1 + w_2 v_2 + w_3 v_3 \quad (2)$$

where

$y \equiv$  income.

An increase in the tariff  $t$  lowers import and leads to adjustments in outputs and domestic factor prices. The resulting adjustment in income depends on factor intensity and substitution as well as prices, domestic factor endowments, and the tariff rate.

## 2. Production in general equilibrium with an imported factor

This section develops the comparative static model of production with an imported factor of production. Imported energy is utilized in the two sectors according to  $v_1 = \sum_j a_{1j} x_j$  for  $j = 1, 2$  where  $a_{1j}$  is the flexible cost minimizing input per unit of output. Adjustments occur according to  $dv_1 = \sum_j (a_{1j} dx_j + x_j da_{1j})$ . Letting the prime ' denote percentage change,

$$v_1' = \sum_j \lambda_{1j} (a_{1j}' + x_j'), \quad (3)$$

where

$a_{1j} \equiv$  input per unit of output

$\lambda_{1j} \equiv a_{1j} x_j / v_1 =$  industry share of factor 1 in sector  $j$ .

Homothetic production implies unit inputs  $a_{1j}$  are functions of factor prices only. Industry shares of each factor sum to one across goods,  $\sum_j \lambda_{ij} = 1$ .

The imported factor price  $f_1 \equiv (1 + t)w_1$  changes with a tariff according to  $df_1 = w_1 dt$  assuming the exogenous world price  $w_1$  is constant, leading to

$$\tau' \equiv df_1/f_1 = dt/(1 + t). \quad (4)$$

where

$\tau'$   $\equiv$  percentage change in the domestic price of factor 1 due to a tariff.

Substitution elasticities reflect adjustments in the factor mix due to changing factor prices.

The cross price substitution elasticity of imported factor 1 relative to the price of domestic factor  $i$  is the industry share weighted cross price elasticity,  $\sigma_{1i} \equiv \sum_j \lambda_{1j} (a_{1j}'/w_i')$ .

The three own substitution elasticities are negative due to Shephard's lemma and concavity of the cost functions. Linear homogeneity implies elasticities for each input  $k$  sum to zero across changes in factor prices,  $\sum_i \sigma_{ki} = 0$ . In practice, cross price elasticities are estimated from production or cost functions and the own elasticity is derived.

Substitution between two inputs implies positive cross price elasticity. With three factors, one pair may be complements with a negative cross price elasticity. Concavity of the cost function in factor prices requires positive principle minors of the substitution matrix with own effects outweighing cross effects,  $\sigma_{ii}\sigma_{kk} - \sigma_{ik}\sigma_{ki} > 0$  for  $i, k = 1, 2, 3$ .

The cost minimizing input of the import adjusts according to  $a_{1j}' = \sigma_{12}w_2' + \sigma_{13}w_3' + \sigma_{11}\tau'$  expanding import adjustment in (3) to

$$v_1' = \sigma_{12}w_2' + \sigma_{13}w_3' + \sigma_{11}\tau' + \sum_j \lambda_{1j}x_j', \quad (5)$$

where

$\sigma_{1i}$   $\equiv$  substitution elasticity of factor 1 relative to price of factor  $i$ .

Adjustments to changes in domestic factor endowments  $v_2$  and  $v_3$  similar to (5) are included in the comparative static system (8).

Revenue is paid to the factors of production in each sector according to  $p_j x_j = (1 + t)w_1 v_{1j} + w_2 v_{2j} + w_3 v_{3j}$  for  $j = 1, 2$  from (1). Divide by  $x_j$  to find pricing conditions linking goods and factors,  $p_j = (1 + t)w_1 a_{1j} + w_2 a_{2j} + w_3 a_{3j}$ . Differentiate to find  $dp_j = w_1 a_{1j} dt + a_{2j} dw_2 + a_{3j} dw_3 + [(1 + t)w_1 da_{1j} + w_2 da_{2j} + w_3 da_{3j}]$ . The bracketed expression disappears due to the cost minimizing envelope property leading to

$$p_j' = \theta_{1j} \tau' + \theta_{2j} w_2' + \theta_{3j} w_3', \quad (6)$$

where

$$\theta_{ij} \equiv a_{ij} w_i / p_j = \text{factor } i \text{ share in revenue of good } j.$$

Factor shares of each good sum to one across factors due to competitive pricing,  $\sum_i \theta_{ij} = 1$ .

Income  $y$  expressed in terms of tariff revenue and factor payments in (2) changes according to

$$y' = \varphi_1(v_1' + T\tau') + \varphi_2(v_2' + w_2') + \varphi_3(v_3' + w_3'), \quad (7)$$

where

$$T \equiv (1 + t)/t$$

$$\varphi_1 \equiv tw_1 v_1 / y = \text{the imported factor income share}$$

$$\varphi_k \equiv w_k v_k / y = \text{income share of domestic factor } k = 2, 3.$$

Combine conditions for employment in (5), competitive pricing in (6), and income in (7) into the comparative static system (8) with exogenous variables on the right,

$$\begin{pmatrix}
-1 & \sigma_{12} & \sigma_{13} & \lambda_{11} & \lambda_{12} & 0 \\
0 & \sigma_{22} & \sigma_{23} & \lambda_{21} & \lambda_{22} & 0 \\
0 & \sigma_{32} & \sigma_{33} & \lambda_{31} & \lambda_{32} & 0 \\
0 & \theta_{21} & \theta_{31} & 0 & 0 & 0 \\
0 & \theta_{22} & \theta_{32} & 0 & 0 & 0 \\
-\varphi_1 & -\varphi_2 & -\varphi_3 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
v_1' \\
w_2' \\
w_3' \\
x_1' \\
x_2' \\
y'
\end{pmatrix}
=
\begin{pmatrix}
-\sigma_{11}\tau' \\
v_2' - \sigma_{21}\tau' \\
v_3' - \sigma_{31}\tau' \\
p_1' - \theta_{11}\tau' \\
p_2' - \theta_{12}\tau' \\
\varphi_2 v_2' + \varphi_3 v_3' + \varphi_1 T\tau'
\end{pmatrix} \quad (8)$$

Factor intensity is critical to the comparative static adjustments. Assume the factor intensity

$$\theta_{11}/\theta_{12} > \theta_{21}/\theta_{22} > \theta_{31}/\theta_{32}, \quad (9)$$

with the import the most intensive or extreme factor for good 1, domestic factor  $v_3$  extreme in good 2 production, and  $v_2$  the middle factor. Define an intensity term between factors 1 and 2 as  $\theta^{12} \equiv \theta_{11}\theta_{22} - \theta_{12}\theta_{21} > 0$  with similar positive intensity terms  $\theta^{23}$  and  $\theta^{13}$ . Factor intensity is also reflected by industry shares in the positive terms  $\lambda^{12}$ ,  $\lambda^{13}$ , and  $\lambda^{23}$ .

The comparative static model solves for the effects of changes in an exogenous variable in the right hand vector on the endogenous variables with Cramer's rule. The negative determinant of the system matrix is  $\Delta = -\theta^{23}\lambda^{23}$ .

Assume energy is the extreme factor in export production, labor in import competing production, and capital the middle factor as in (9). Thompson (1983) shows an increase in the international price of the imported factor lowers the import and at least one output, raises the wage, and lowers the capital return given the present factor intensity. The production frontier is concave in the relative price of outputs. Factor import cannot be positively related to prices of both goods. An increase in the capital endowment raises factor import and export production, and lowers import competing production. An increase in the labor endowment has opposite effects. The following sections extend these results to include the effect of a factor tariff on income.

### 3. The effects of a factor tariff

A tariff lowers import in the general equilibrium system (8) according to

$$v_1'/\tau' = -\Delta_{32}/\Delta < 0, \quad (10)$$

where  $\Delta_{32}$  is the determinant of the model with three domestic factors. Neoclassical concavity and cost minimization imply  $\Delta_{32} < 0$  as shown by Chang (1979) and Thompson (1985). The implication of (10) is that demand for the import slopes downward allowing all markets in the economy to adjust.

The effects of a tariff on domestic factor prices depend only on factor intensity,

$$\begin{aligned} w_2'/\tau' &= -\theta^{13}/\theta^{23} < 0 \\ w_3'/\tau' &= \theta^{12}/\theta^{23} > 0, \end{aligned} \quad (11)$$

where the intensity terms are positive due to (9). The wage  $w_3$  rises with a tariff as capital is released from export production increasing the marginal product of labor in the import competing sector. The return  $w_2$  to middle factor capital falls as it is released from export production. If imported energy were the middle factor, both domestic factor prices would fall.

Output adjustments on the shrinking production frontier are

$$\begin{aligned} x_1'/\tau' &= (\lambda_{12}\sigma_3 - \lambda_{22}\sigma_4)/\Delta \\ x_2'/\tau' &= (\lambda_{21}\sigma_4 - \lambda_{11}\sigma_3)/\Delta, \end{aligned} \quad (12)$$

where  $\sigma_3 \equiv (\theta^{13} + \theta^{23})\sigma_{21} + \theta\sigma_{23}$ ,  $\sigma_4 \equiv (\theta^{12} - \theta^{23})\sigma_{31} + \theta\sigma_{32}$ , and  $\theta \equiv (\theta^{12} + \theta^{13})$ . Thompson (1983) shows at least one of these output adjustment must be negative.

Income adjusts according to

$$y'/\tau' = \varphi_1(T + v_1'/\tau') + \varphi_2(w_2'/\tau') + \varphi_3(w_3'/\tau'). \quad (13)$$

The first term in (13) reflects the ambiguous change in tariff revenue  $tw_1v_1$  that would be summarized by the import elasticity. The second and third terms are weighted effects of

domestic factor price adjustments in (11). An increase in income is favored by a lower tariff level or higher  $T$ , inelastic import demand, a higher labor income share  $\varphi_3$ , stronger import intensity  $\theta^{12}$  relative to middle factor 2, and stronger intensity between the domestic factors  $\theta^{23}$ .

To illustrate the potential of a tariff to raise income, consider an economy facing unit world prices  $w_1 = p_1 = p_2 = 1$  with unit factor endowments  $v_2 = v_3 = 1$ . Equilibrium levels are outputs  $x_1 = 2.50$  and  $x_2 = 0.83$ , domestic factor prices  $w_2 = 0.77$  and  $w_3 = 1.10$ , and energy import  $v_1 = 1.33$ . Assume tariff rate  $t = 0.10$  generating tariff revenue  $tw_1v_1 = 0.13$ . The value of output  $x = 3.33$  in (1) less import spending equals income  $y = 2.00$  in (2).

The factor share and industry share matrices are

$$\begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \\ \theta_{31} & \theta_{32} \end{pmatrix} = \begin{pmatrix} 0.55 & 0.11 \\ 0.23 & 0.23 \\ 0.22 & 0.66 \end{pmatrix} \quad \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ \lambda_{31} & \lambda_{32} \end{pmatrix} = \begin{pmatrix} 0.94 & 0.06 \\ 0.75 & 0.25 \\ 0.50 & 0.50 \end{pmatrix}, \quad (14)$$

consistent with the factor intensity in (9). Cost minimizing inputs  $a_{ij}$  equal the derived factor shares in (14).

Assume the substitution elasticities

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} -0.9 & -0.6 & 1.5 \\ -0.2 & -0.3 & 0.5 \\ 0.2 & 0.2 & -0.4 \end{pmatrix}, \quad (15)$$

satisfying concavity conditions with energy and capital technical complements. These elasticities are consistent with the static equilibrium and would be derived as industry share weighted averages of substitution elasticities in each sector. Sector elasticities in practice are derived from estimates of production or cost functions. The elasticities in (15) can be derived in the present specification from the quadratic cost functions in the two sectors  $c_1 = -w_1^2 - w_2^2 - w_3^2 + 0.86w_1w_2 + 1.72w_1w_3 + 0.89w_2w_3$  and  $c_2 = -w_1^2 - w_2^2 - w_3^2 + 0.29w_1w_2 + 1.72w_1w_3 + 1.41w_2w_3$ .

Solving the resulting comparative static model (8) for a change in the factor tariff, energy import adjusts according to  $v_1'/\tau' = -0.33$ . The elastic domestic factor price adjustments are  $w_2'/\tau' = -3.35$  and  $w_3'/\tau' = 1.00$ . An increase in the tariff induces substitution toward labor, raising the wage  $w_3$  as substitution away from complementary capital strongly lowers the capital return  $w_2$ . Elastic adjustments in outputs  $x_1'/\tau' = -3.48$  and  $x_2'/\tau' = 5.22$  shift production toward import competition favoring its extreme factor labor.

Income increases with the tariff according to  $y'/\tau' = 0.05$  due to elastic adjustments in outputs and domestic factor prices and the inelastic decrease in imported energy.

## 5. Conclusion

The effects of a factor tariff in a competitive small open economy depend on factor intensity substitution, factor shares of income, and the state of the economy. A factor tariff lowers import spending and shrinks the production frontier. When there are three or more factors of production, a potential increase in income arises due to the flexibility of substitution and outputs.

In the present model with three factors and two goods, a tariff on imported energy favors the import competing sector and raises the price of its intensive factor. Export production falls as does the return to the middle intensity factor. Under some conditions, factor import spending falls more than the reduced value of output resulting in increased income.

If imported energy is a complement with capital, the higher domestic price of energy strongly reduces capital demand and its return. Strong substitution of labor for energy generates a large increase in labor demand. The inelastic reduction in energy import coupled with elastic adjustments in outputs and domestic factor prices account for an increase in income.

Other arguments favoring a tariff can be mentioned. For a large economy, a tariff lowers the international price raising the possibility of a Metzler (1949) paradox with a lower price including the tariff. Thompson (2016) shows a tariff can raise income in an economy that has import competing domestic factor supply. Facing a foreign monopoly, a tariff transforms foreign profit into tariff revenue. Finally, a tariff reduces any externalities associated with the imported factor.

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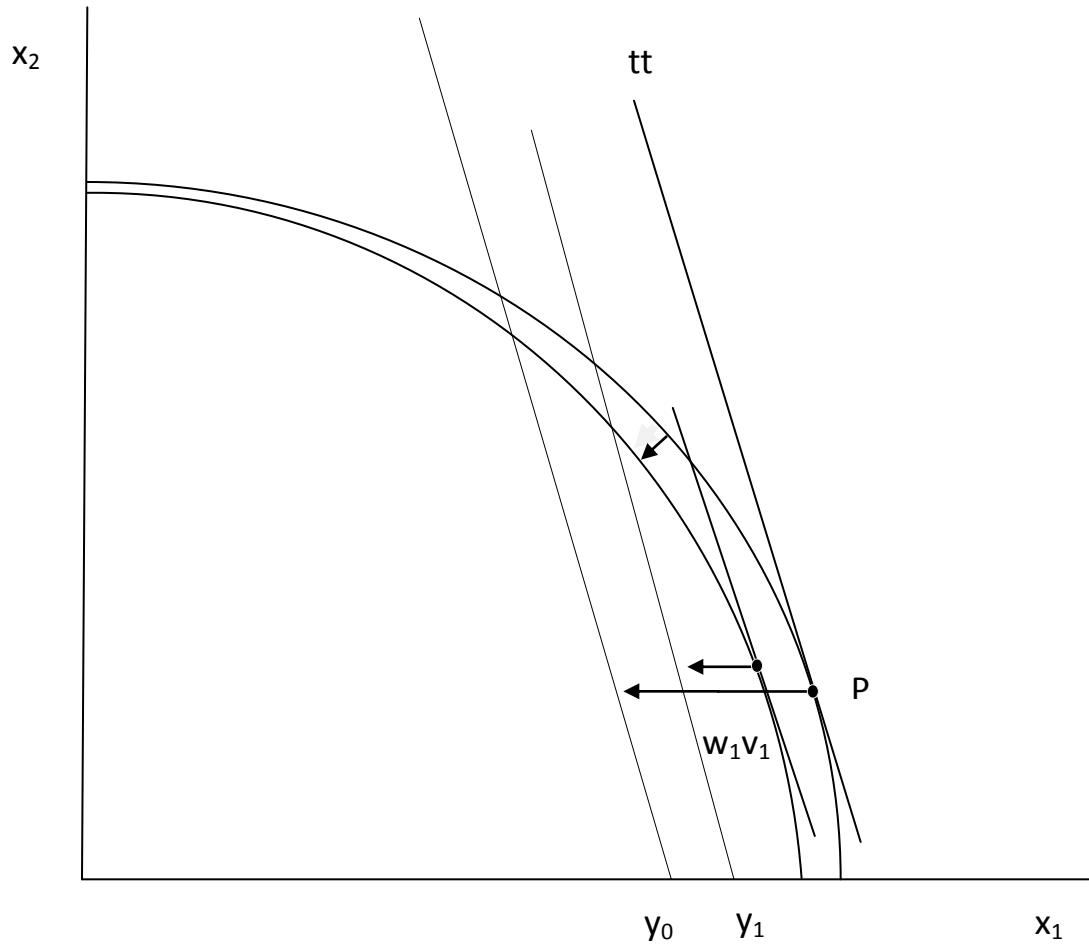


Figure 1. A factor tariff and income